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ON THE KATO-ROSENBLUM THEOREM

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The Kato-Rosenblum Theorem has no straightforward generalization to operators with non-absolutely continuous spectra. For example, if A is a bounded selfadjoint operator such that the singular continuous parts of H and $H + A$ are unitarily equivalent for every selfadjoint operator H, then $A = 0$.

1. Introduction. The classical theorem of Kato and Rosenblum (1957) asserts the invariance of absolutely continuous parts under tracse class perturbations. [**5**, p. 540; **6**, p. 26]

THEOREM (*Kato-Rosenblum*). *If H and A are selfadjoint, and A is trace class, then the absolutely continuous parts of H and $H + A$ are unitarily equivalent.*

It is notable that the theorem gives a unitarily invariant condition on the perturbation A alone, and that Lebesgue measure plays a distinguished role.

That the trace condition cannot be radically improved, follows from the Weyl-von Neumann theorem [**5**, p. 523], which states that given any selfadjoint operator H, there is a selfadjoint perturbation A of arbitrarily small Hilbert-Schmidt norm, such that $H + A$ has pure point spectrum—a phenomenon often termed *curdling*. Moreover, according to Kuroda, the Hilbert-Schmidt norm may be replaced by any cross-norm *except* the trace norm. [**5**, p. 525]

For singular measures, there are a few, largely negative, results. Donoghue [**2**], following earlier work of Aronszajn, gave examples in which a purely singular continuous spectrum is curdled by a perturbation of rank one. He also obtained the following result, which we shall use [**2**, p. 565; **4**, Cor. 1].

THEOREM. (*Donoghue*). *Let H be selfadjoint and $A = c\langle\,\cdot\,,\phi\rangle\phi$ where ϕ if cyclic for H and c is real and non-zero. Then the singular parts of H and $H + A$ are supported on disjoint sets (i.e. are mutually singular).*

A generalization was proved in [**4**].