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LIE'S FUNDAMENTAL THEOREMS FOR LOCAL ANALYTICAL LOOPS

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A central piece of classical Lie theory is the fact that with each local Lie group, a Lie algebra is associated as tangent object at the origin, and that, conversely and more importantly, every Lie algebra determines a local Lie group whose tangent algebra it is. Up to equivalence of local groups, this correspondence is bijective.

Attempts at the development of a Lie theory for analytical loops have not been entirely satisfactory in this direction, since they relied more or less on certain associativity assumptions. Here we associate with an arbitrary local analytical loop a unique tangent algebra with a ternary multiplication in addition to the standard binary one, and we call this algebra an Akivis algebra. Our main objective is to show that, conversely, for every Akivis algebra there exist many inequivalent local analytical loops with the given Akivis algebra as tangent algebra. We shall give a good idea about the degree of non-uniqueness. It is curious to note that, on account of this non-uniqueness, the construction is more elementary than in the case of analytical groups.

The First and Third Fundamental Theorems of Sophus Lie assert the following statements (Cf. [**18**], IV, Kap. 15, pp. 365–404):

THE FIRST THEOREM. *Every local analytical group determines on its tangent vector space at the identity the structure of a unique Lie algebra with the Lie bracket given by the formula*

$$[x, y] = \lim_{t \to 0} t^{-2}\left(\frac{tx \circ ty}{ty \circ tx}\right) \tag{1}$$

where the local group operation $\circ$ is transported into the tangent space, and where g/h is written for $g \circ h^{-1}$.

THE THIRD THEOREM. *If L is a finite dimensional real Lie algebra, then there exists a local analytical group whose tangent Lie algebra according to the First Theorem is isomorphic to L. Moreover, local analytical groups with this property are locally isomorphic.*