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SPACE CURVES THAT INTERSECT OFTEN

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In intersection theory one tries to understand $X \cap Y$ in terms of information about how X and Y lie in an ambient variety Z. When the sum of the codimensions of X and Y in Z exceeds the dimension of Z not much is known in this direction. The purpose of this note is to provide some results in perhaps the simplest nontrivial case of this—that of curves in $\mathbf{P}^3$ (projective three space). A weaker result for $\mathbf{P}^n$ is also obtained. We work over any fixed algebraically closed field of arbitrary characteristic.

(1) THEOREM. *Let X of degree d and Y of degree e be two distinct reduced, irreducible curves in $\mathbf{P}^3$ neither of which is contained in a hyperplane. Assume $d \leq e$. Let m be the number of points in $X \cap Y$ (not counting multiplicity). Then*:

(i) $m \leq (d-1)(e-1)+1$

(ii) *If $m = (d-1)(e-1)+1$ then there exists a quadric hypersurface Q containing $X \cup Y$. If furthermore $d \geq 4$ then Q is smooth and on Q X has type $(d-1, 1)$ and Y has type $(1, e-1)$.*

(iii) *If $d \geq 4$ and $m \geq (d-2)e+1$ then there exists a smooth quadric Q containing $X \cup Y$.*

The key to the proof of this theorem will be a study of the ideal of the curve X. Results of [**GLP**] will be crucial.

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Before starting the proof of (1) we quote results from other sources that will be needed.

(2) DEFINITION ([**GLP**], p. 491). Let $X \subset \mathbf{P}^r$ be a reduced curve. For a given integer $n \geq 0$ we say X satisfies property (C_n) if X is cut out in $\mathbf{P}^r$ by hypersurfaces of degree n, and the homogeneous ideal of X is generated in degrees greater than or equal to n by its component of degree n.