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CONSTRUCTIONS OF TWO-FOLD BRANCHED COVERING SPACES

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By equivariantly pasting together exteriors of links in S^3 that are invariant under several different involutions of S^3, we construct closed orientable 3-manifolds that are two-fold branched covering spaces of S^3 in distinct ways, that is, with different branch sets. Sufficient conditions are given to guarantee when the constructed manifold M admits an induced involution, h, and when $M/h \cong S^3$. Using the theory of characteristic submanifolds for Haken manifolds with incompressible boundary components, we also prove that doubles, $D(K,\rho)$, of prime knots that are not strongly invertible are characterized by their two-fold branched covering spaces, when $\rho \neq 0$. If, however, K is strongly invertible, then the manifold branch covers distinct knots. Finally, the authors characterize the type of a prime knot by the double covers of the doubled knots, $D(K;\ \rho,\eta)$ and $D(K^*;\ \rho,\eta)$, of K and its mirror image K^* when ρ and η are fixed, with $\rho \neq 0$ and $\eta \in \{-2,2\}$.

With each two-fold branched covering map, $p\colon M^3 \to N^3$, there is associated a PL involution, $h\colon M \to M$, that induces p. There can, however, be other PL involutions on M that are not equivalent to h, but nevertheless are covering involutions for two-fold branched covering maps of M (cf. **[BGM]**). Our purpose, in this paper, is to introduce ways of detecting such involutions and controlling their number. We begin with compact 3-manifolds with several obvious PL involutions.

An oriented link $\tilde{L}$ in M is *2-symmetric*, if $N^3 \cong S^3$ and if $h(\tilde{L}) = \tilde{L}$. In §1, we give examples of knots and links in S^3 that are 2-symmetric in two or more ways; for example, a trefoil knot is both strongly invertible and periodic (definitions in §1). In §2, we paste the exteriors, $E(\tilde{L})$ and $E(\tilde{L}')$, of 2-symmetric links, $\tilde{L}$ and $\tilde{L}'$, together along a torus-boundary component of each exterior; Proposition 2.1 gives the pasting instructions f that must be followed in order for the involutions, h and h', of $E(\tilde{L})$ and $E(\tilde{L}')$ to extend to an involution h_f of $E(\tilde{L}) \cup_f E(\tilde{L}')$. Theorems 2.2 and 2.3 allow us to conclude, under fairly relaxed conditions, that the orbit space of h_f is S^3.