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SCHRÖDINGER OPERATORS WITH A NONSPHERICAL RADIATION CONDITION

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The Schrödinger operators with potentials $p(x)$ which do not necessarily converge to a constant at infinity will be discussed. The potential $p(x) = x_1/|x|$, $x = (x_1, x_2, \ldots, x_n) \in \mathbf{R}^N$, is an example. The radiation condition associated with such Schrödinger operators is shown to have the form $\nabla u - i\sqrt{\lambda}(\nabla R)u =$ small at infinity, where $R = R(x, \lambda)$ is a solution of the eikonal equation $|\nabla R|^2 = 1 - p(x)/\lambda$. This radiation condition is "nonspherical" in the sense that ∇R is not proportional to the vector $\tilde{x} = x/|x|$ in general. The limiting absorption principle will be obtained using a priori estimates for the radiation condition.

Introduction. Let us consider the inhomogeneous Schrödinger equation

$$(0.1)\qquad (T - \lambda)u = -\sum_{j=1}^{N} D_j^2 u + V(x)u - \lambda u = f \quad \text{in } \mathbf{R}^N,$$

where $D_j = \partial/\partial x_j + ib_j(x)$ with the "magnetic potentials" $b_j(x)$, λ is a positive number, the "potential" $V(x)$ is a real-valued function on $\mathbf{R}^N$ and $f(x)$ is a given function. In this paper we are going to consider a class of potentials $V(x)$ which contains potentials $V(x)$ such that $V(x) = O(1)$ and $\partial V/\partial x_j = O(|x|^{-1})$ at $x = \infty$. One example of such a function is $V(x) = x_1/|x|$ where x_1 is the first coordinate of $x = (x_1, x_2, \ldots, x_N) \in \mathbf{R}^N$. We shall study the limiting absorption principle and the unique existence of the solution $u = u(\lambda, f)$ of the equation (0.1) introducing a "nonspherical" radiation condition

$$(0.2)\quad \left(D_j - i\sqrt{\lambda}\,\beta_j\right)u(x) \quad \text{is small at } x = \infty \qquad (j = 1, 2, \ldots, N).$$

Condition (0.2) is nonspherical in the sense that $\beta = (\beta_1, \beta_2, \ldots, \beta_N)$ is the outward normal of a surface which is not a sphere in general, whereas it seems that the outward normal $\tilde{x} = x/|x|$ of a sphere always appeared in the radiation conditions which were used up to now for various types of Schrödinger operators.

Let us first assume that $V(x)$ becomes small at $x = \infty$. Then the unique existence of the solution $u = u(\lambda, f)$ of the equation $(0, 1)$ with