PACIFIC JOURNAL OF MATHEMATICS
Vol. 126, No. 2, 1987

DERIVATIONS ON THE LINE AND FLOWS ALONG ORBITS

C. J. K. BATTY

The closure of the derivation λD: $C_c^1(\mathbb{R}) \to C_0(\mathbb{R})$ defined by $(\lambda D)(f) = \lambda f'$, where λ: $\mathbb{R} \to \mathbb{R}$ is continuous, generates a C_0-group on $C_0(\mathbb{R})$ (corresponding to a flow on $\mathbb{R}$) if and only if $1/\lambda$ is not locally integrable on either side of any zero of λ or at $\pm\infty$.

If S is a flow on a locally compact, Hausdorff, space X with fixed point set X_S^0, δ_S is the generator of the induced action on $C_0(X)$, λ: $X \setminus X_S^0 \to \mathbb{R}$ is continuous, and bounded on sets of low frequency under S, and $t \to \lambda(S_t\omega)^{-1}$ is not locally integrable on either side of any zero or at $\pm\infty$, then the flows along the orbits of S form a flow on X whose generator acts as $\lambda\delta_S$.

1. Introduction. Let S be a flow on a locally compact, Hausdorff, space X, and δ_S be the generator of the associated one-parameter group of *-automorphisms of $C_0(X)$, the commutative C^*-algebra of continuous complex-valued functions on X which vanish at infinity. Thus

$$\delta_S f = \lim_{t\to 0} t^{-1}(f \circ S_t - f)$$

whenever the limit exists (pointwise, and hence uniformly) and defines a function in $C_0(X)$. Let $\mathcal{D}_S^\infty = \bigcap_{n\geq 1}\mathcal{D}(\delta_S^n)$. Then $\mathcal{D}_S^\infty$ is a dense *-subalgebra of $C_0(X)$. If δ: $\mathcal{D}_S^\infty \to C_0(X)$ is a *-derivation, then there is a function λ: $X \to \mathbb{R}$ such that

$$\delta f = \lambda \delta_S f \qquad (f \in \mathcal{D}_S^\infty)$$

[**1**]. The function λ may be chosen arbitrarily on the fixed point set X_S^0:

$$\begin{aligned} X_S^0 &= \{\omega \in X\colon S_t\omega = \omega \text{ for all } t\} \\ &= \{\omega \in X\colon \delta_S f(\omega) = 0 \text{ for all } f \text{ in } \mathcal{D}_S^\infty\}, \end{aligned}$$

and we shall always assume that $\lambda = 0$ on X_S^0. However, λ is uniquely determined and continuous on $X \setminus X_S^0$, and satisfies a bound of the form

$$(*) \qquad |\lambda(\omega)| \leq c(1 + \nu(\omega)^n) \qquad (\omega \in X \setminus X_S^0)$$

for some constant $c \geq 0$, and integer $n \geq 0$, where $\nu(\omega)$ is the frequency of ω, so

$$\nu(\omega)^{-1} = \inf\{t > 0\colon S_t\omega = \omega\}$$

($\nu(\omega) = 0$ if ω is aperiodic) (see [**4**]).