

ANOTHER CHARACTERIZATION OF $AE(0)$ -SPACES

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We prove that a space X is an absolute extensor for the class of all zero-dimensional spaces if and only if X is an upper semi-continuous compact-valued retract of a power of the real line.

1. Introduction. Dugundji spaces were introduced by Pelczynski [5]. Later Haydon [4] proved that the class of Dugundji spaces coincides with the class of all compact absolute extensors for zero-dimensional compact spaces (briefly, $AE(0)$). After Haydon's paper, compact $AE(0)$ -spaces have been extensively studied (see Ščepin's review [9]); let us note the following result of Dranishnikov [3]: a compact X is an $AE(0)$ -space if and only if for every embedding of X in a Tychonoff cube I^τ there exists an upper semi-continuous compact-valued (br. usco) mapping r from I^τ to X such that $r(x) = \{x\}$, for each $x \in X$ (such a usco mapping will be called a usco retraction).

Chigogidze [2] extended the notion of $AE(0)$ from the class of compact spaces to that of completely regular spaces and gave a characterization of such $AE(0)$ -spaces.

The aim of the present paper is to give another characterization of completely regular $AE(0)$ -spaces which is similar to the above mentioned result of Dranishnikov. We prove that $X \in AE(0)$ iff X is a usco retract of R^τ for some τ , where R is the real line with the usual topology. Our technique is different from Dranishnikov's.

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2. Notations and terminology. All spaces considered are completely regular and all single-valued mappings are continuous. A set-valued mapping r from X to Y is called upper semi-continuous (br. u.s.c.) if the set $r^\#(U) = \{x \in X : r(x) \subset U\}$ is open in X whenever U is open in Y . We say that a usco mapping r is minimal if every usco selection for r coincides with r . It follows from the Kuratowski-Zorn lemma that every usco mapping has a minimal usco selection.

A mapping f from Y to X , where $Y \subset Z$, is called Z -normal if, for every continuous function g on X , the function $g \circ f$ is continuously extendable to Z . A space X is called an absolute extensor for zero-dimensional spaces [2], if every Z -normal mapping f from Y to X , where $Y \subset Z$