PACIFIC JOURNAL OF MATHEMATICS
Vol. 127, No. 1, 1987

SURGERY ON A CLASS OF PRETZEL KNOTS

WILLIAM ORTMEYER

Any closed 3-manifold M may be thought of as a union of 3-cells. Any covering space of M is made up of copies of these 3-cells with boundaries locally identified as in M. Covers of M may be built by piecing together these balls. This paper develops a method to piece together the universal cover of manifolds obtained from the 3-sphere by surgery on a class of pretzel knots in such a way that the cover can be shown to be R^3.

1. It has been shown [**4**, **6**, **14**] that any connected orientable 3-manifold may be constructed by surgery along a finite number of knots in S^3, that is, by removing tubular neighborhoods of one or more smooth knots and sewing them back in differently. In this paper we study the 3-manifolds obtained by surgery on certain pretzel knots [**2**, **8**] by showing that the universal cover of such manifolds is R^3. We thus show that these pretzel knots satisfy several hoped for conjectures (property P: nontrivial surgery never yields a simply connected manifold, and property R: surgery never yields a manifold with fundamental group Z).

The manifolds obtained by surgery on pretzel knots have been studied algebraically. In fact, it is known that all pretzel knots (except, of course, the unknot, which has several pretzel knot representations) have property R [**5**, **7**]. The property P question has been answered affirmatively for certain classes of pretzel knots [**1**, **9**, **10**, **11**], but the question remains open in general. We will take a different, geometric approach to the problem. In §2 we introduce the basic concepts we will use to build the covering spaces of the manifolds, as well as prove a lemma which identifies the covering space in certain situations. In §3 we apply the ideas of §2 to the surgery manifolds of certain pretzel knots and construct their universal covering spaces.

2. In this section we develop concepts which allow us, in certain cases, to build and identify covering spaces inductively. The central idea is "precover" which isolates properties a space needs in order to be part of a covering space (i.e., an incompletely built covering space).

DEFINITION 2.1. Let $p\colon \tilde{X} \to X$ be a map. $(\tilde{X}, p, X)$ is a *precover* if for each $x \in X$ there is a connected open set U containing x such that

(i) $p^{-1}(U) = \bigcup S_a$ where the S_a's form a collection (nonempty if x is in $p(\tilde{X})$) of mutually disjoint connected open sets in $\tilde{X}$, each containing