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BOUNDARY BEHAVIOR OF HOLOMORPHIC FUNCTIONS IN THE BALL

JACOB BURBEA

A description of the boundary behavior of functions belonging to certain Sobolev classes of holomorphic functions on the unit ball B_n of $\mathbf{C}^n$ is given in terms of bounded and vanishing mean oscillation. In particular, it is shown that the boundary values of any holomorphic function on B_n, whose fractional derivative of order n/p belongs to the Hardy class $H^p(B_n)$, have vanishing mean oscillation provided $0 < p \leq 2$.

Introduction. Let $B = B_n$ be the unit ball in $\mathbf{C}^n$ and let $H(B)$ denote the space of all holomorphic functions on B. By R we denote the *radial derivative operator* $R = \sum z_j \partial_j$ where for $z = (z_1, \ldots, z_n) \in \mathbf{C}^n$, $\partial_j = \partial/\partial z_j$ $(j = 1, \ldots, n)$, and we let $D_l = l + R$, with $D = D_1$, for any $l \in \mathbf{C}$. For any monomial $z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n}$, $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbf{Z}_+^n$, we have $D_l^s z^\alpha = (|\alpha| + l)^s z^\alpha$ for any $s \in \mathbf{Z}_+$, where $|\alpha| = \alpha_1 + \cdots + \alpha_n$, which shows that for any $l > 0$, $s \in \mathbf{R}$ and $f \in H(B)$, the *fractional derivative* $D_l^s f$ of f, of order s, is well-defined and is in $H(B)$. Let $H^p = H^p(B)$, $0 < p \leq \infty$, denote the usual Hardy class of functions in $H(B)$. The *Hardy-Sobolev class* $H_s^p = H_s^p(B)$ $(0 < p \leq \infty,\ s \in \mathbf{R})$ is defined as the space of all f in $H(B)$ whose fractional derivative $D^s f$ is in H^p, and thus $H_0^p = H^p$.

In the one-dimensional case $(n = 1)$, most of the main properties of these H_s^p spaces were investigated early by Privalov in 1918, by Hardy and Littlewood in 1932, and by Smirnov in 1932 (see the references of [**7**]). The question of extending these results to the higher dimensional case $(n \geq 2)$ has been considered previously by Graham [**8**, **9**] and Krantz [**10**], and, quite recently, by Beatrous and Burbea [**3**], where these spaces are viewed as a special case of a larger family of Sobolev spaces of holomorphic functions on B. The following result, among other things, appears in [**3**]:

THEOREM 1.1. *Let* $0 < p \leq \infty$ *and* $s \geq 0$.

(i) *If* $s > n/p$ *then* H_s^p *is contained in the Lipschitz class* $\Lambda_{s-n/p}(B)$;

(ii) *If* $0 \leq s < n/p$ *then* H_s^p *is contained in the Hardy class* $H^{pn/(n-ps)}(B)$;