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A GEOMETRIC FUNCTION DETERMINED BY EXTREME POINTS OF THE UNIT BALL OF A NORMED SPACE

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A geometric function, which measures the relative distance of a vector to an extreme point of the unit ball of a normed space, is defined. This function is calculated explicitly for certain classical function and sequence spaces. Radial limits and continuity properties of this function are investigated and some applications are given.

Introduction. There are many normed spaces X, which are geometrically very different, whose closed unit balls have the following geometric property, called the λ-property, in common: each member x of the unit ball is a convex combination of an extreme point e of the unit ball and a vector y, where $\|y\| \leq 1$ and e is assigned a positive weight. If we vary e and y, looking for the "largest possible" weight in such a representation of x, we obtain a geometric function of x, called the λ-function, which measures how close x is to being an extreme point of the unit ball. In Section 1, we make these ideas more precise and calculate explicit formulas for the λ-function for the classical spaces $C_X(T)$, $l_1(X)$, $l_\infty(X)$ and $c(X)$. It is also shown when the "largest possible" weight is attained in these spaces. Section 2 investigates continuity properties of the λ-function. These include existence of radial limits (Theorem 2.2) and Lipschitz properties (Corollaries 2.8 and 2.9). In Section 3, it is shown how the uniform λ-property is related to uniformly convergent series expansions of vectors in terms of infinite convex combinations of extreme points of the unit ball (Theorem 3.1). Local boundedness of the λ-function away from zero (Theorem 3.5) is also discussed. Section 4 contains a list of questions and open problems.

0. Notation. If X is a normed space, the closed unit ball, open unit ball and unit sphere will be denoted by B_X, U_X and S_X, respectively. The symbols $l_1(X)$, $l_\infty(X)$ and $c(X)$ denote the spaces of all X-valued sequences $x = (x_n)$ which are absolutely summable, bounded and convergent, respectively. $l_1(X)$ is endowed with the norm $\|x\| = \sum_{n=1}^{\infty}\|x_n\|$, while the norm in $l_\infty(X)$ and $c(X)$ is given by $\|x\| = \sup_n\|x_n\|$. If T is a