PACIFIC JOURNAL OF MATHEMATICS
Vol. 128, No. 1, 1987

ON A COHOMOLOGY THEORY BASED ON HYPERFINITE SUMS OF MICROSIMPLEXES

RADE T. ŽIVALJEVIĆ

In this note we investigate a cohomology theory $H^{\#}(X, G)$, defined by M. C. McCord, which is dual to a homology theory based on hyperfinite chains of miscrosimplexes. We prove that if X is a locally contractible, paracompact space then $H^{\#}(X, G) \simeq H_{\check{c}}^{\#}(X, \mathrm{Hom}({}^{*}Z, G))$ where $H_{\check{c}}^{\#}$ is the Čech theory. Nonstandard analysis, particularly the Saturation Principle, is used in this proof in essential way to construct a fine resolution of the constant sheaf $X \times \mathrm{Hom}({}^{*}Z, Z)$. This gives a partial answer to a question of McCord. Subsequently, we prove a proposition from which it is deduced that $\mathrm{Hom}({}^{*}Z, Z) = \{0\}$ i.e. $H^{\#}(X, Z) = \{0\}$ if X is paracompact and locally contractible. At the end we briefly discuss a related cohomology theory which is obtained by application of the internal (rather than external) $\mathrm{Hom}(\cdot, G)$ functor.

Introduction. As it is well known, nonstandard or infinitesimal analysis of Abraham Robinson was developed in an attempt to justify usage of infinitesimals and infinite numbers in calculus and other areas of mathematics. In the case of a general topological space (X, τ), a related notion is notion of the monad of $x \in X$, more precisely

$$\mathrm{monad}(x) = \bigcap\{{}^{*}V \mid V \in \tau,\ x \in V\}.$$

Informally, the monad of x is the set of all $y \in {}^{*}X$ which are infinitely close to x. This leads to a precise definition of a microsimplex. A $(n+1)$-tuple $s = (a_0, \ldots, a_n) \in ({}^{*}X)^{n+1}$ is a microsimplex if there exists $x \in X$ such that $\{a_0, \ldots, a_n\} \subset \mathrm{monad}(x)$.

Motivated by Vietoris homology and Alexander-Spanier cohomology, where the notion of a small simplex is used only in an informal sense, M. C. McCord in [7] defined a conceptually clear and technically easy homology theory based on hyperfinite chains of microsimplexes. The proofs are given in such a way that one automatically gets an associated cohomology theory by composing the chain complex functor with (external) functor $\mathrm{Hom}(\cdot, G)$.

At the end of his paper McCord raised three natural questions. The first two were about the relationship of his and Čech homology theory, whereas in the third a similar question is asked for his cohomology theory. The first two questions were answered by S. Garavaglia in [4]. He proved