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SPECIAL GENERATING SETS OF PURELY INSEPARABLE EXTENSION FIELDS OF UNBOUNDED EXPONENT

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The present paper considers the problem of choosing a maximum subfield having a subbasis over K among subextensions of L/K, when L/K is purely inseparable but of unbounded exponent.

Throughout L will be a purely inseparable extension field of a field K of characteristic $p \neq 0$. For the case when L/K is of bounded exponent $e > 0$ Weisfeld [**6**, Theorem 3, p. 442] has shown that among the subfields of L having a subbasis over K there is a maximal subfield with respect to set inclusion. This theorem fails in the unbounded exponent case since such a maximal subfield would not always exist [**6**, p. 442]. An open problem was, therefore, posed in Weisfeld's paper regarding a necessary and sufficient condition for the theorem to hold for extensions L/K of unbounded exponent. The present paper seeks to provide a solution to this problem.

Let M be a given subset of L. The subset M will be said to be in *canonical form* when M is put in the form $M = A_1 \cup A_2 \cup \cdots$ where A_i consists of the elements of M having exponent i over K. M is called a *canonical generating set over* K if M is a minimal generating set for $K(M)$ and when $M = A_1 \cup A_2 \cup \cdots$ in canonical form, then the subsets M_i defined by $M_i = \bigcup_{j=i+1}^{\infty} A_j$, $i = 0, 1, \ldots$, $M_0 = M$, satisfy $M_i^{p^i}$ is a minimal generating set for $K(M^{p^i})/K$. The set M is called a *distinguished subset of* L/K if M is a canonical generating set over K and, for each nonnegative integer n, $K \cap L^{p^n} \subseteq K^p(A_n^p \cup A_{n+1}^p \cup \cdots)$ where $M = A_1 \cup A_2 \cup \cdots$ in canonical form. Finally, M is called a *subbasis over* K if for every finite subset $\{a_1, \ldots, a_r\}$ of M, $K(a_1, \ldots, a_r)$ is the tensor product of the simple extensions $K(a_i)$, $i = 1, \ldots, r$, and when this happens, the extension $K(M)$ is called an extension having a subbasis over K.

The main result is that if L/K is any purely inseparable extension, then L/K has a maximal subfield J having a subbasis over K if and only if L/K has a distinguished subset M.

LEMMA 1. *If L/K has a subbasis, then every subbasis for L/K is distinguished.*