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THE BEST MODULUS OF CONTINUITY FOR SOLUTIONS OF THE MINIMAL SURFACE EQUATION

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We consider the Dirichlet problem for the minimal surface equation on a bounded domain in $\mathbf{R}^n$ which has nonnegative mean curvature. We give a modulus of continuity for the solution u in terms of the modulus of continuity of the boundary values ϕ. The modulus obtained is shown to be best possible.

0. Introduction. We consider the Dirichlet problem for the minimal surface equation in a bounded domain $\Omega \subseteq \mathbf{R}^N$ with boundary values ϕ. We shall assume that $\partial\Omega$ has nonnegative mean curvature and so the existence of a solution u for any continuous function ϕ is known **[JS]**. In this paper we consider the way that the regularity of u depends on the regularity of ϕ. Many results have already been obtained for this problem. For example if $\phi \in C^{k,\alpha}(\partial\Omega)$, $k \geq 1$, then $u \in C^{k,\alpha}(\overline{\Omega})$. (See **[GG]**, **[L1]**, **[GT]**.) If $\phi \in C^{0,1}(\partial\Omega)$ then $u \in C^{0,\alpha}(\overline{\Omega})$ for some $\alpha \in (0,1)$ which depends on the Lipschitz constant of ϕ and the behaviour of the mean curvature of $\partial\Omega$. (See **[G1]** and also **[W1]** where optimal results are obtained for the value of α.) If $\phi \in C^{0,\alpha}(\partial\Omega)$, $0 < \alpha \leq 1$, and the mean curvature of $\partial\Omega$ is strictly positive then $u \in C^{0,\alpha/2}(\overline{\Omega})$ (see **[G1]** and **[L2]**.) More generally if $x_0 \in \partial\Omega$, ϕ satisfies a Hölder condition with exponent α at x_0 and the mean curvature of $\partial\Omega$ is larger than $C|x - x_0|^k$, $C > 0$, near x_0 then u satisfies a Hölder condition at x_0 with exponent $\alpha/(k+2)$. (See **[W1]** and **[W2]**.) However few results, apart from just continuity, have been given in the case $\phi \in C^{0,\alpha}(\partial\Omega)$, $0 < \alpha < 1$, and assuming only nonnegative mean curvature. A modulus of continuity could be found by the method proposed in §13.5 of **[GT]**. The counter-examples of **[W1]** show that in general the solution u will not be Hölder continuous for any exponent β. We shall explicitly find a modulus of continuity for u in this situation and then show that it is the best possible. Actually the results hold, and are presented, for a fairly general modulus of continuity for ϕ rather than just the Hölder condition. Further the results given are local results and so we only need to assume $\partial\Omega$ has nonnegative mean curvature in a neighbourhood of the point under consideration. However in this case