PACIFIC JOURNAL OF MATHEMATICS
Vol. 129, No. 1, 1987

GENERALIZED RIGID ELEMENTS IN FIELDS

KAZIMIERZ SZYMICZEK

Rigid elements in a field, should they exist, have strong influence on structure of the Witt ring of the field. We generalize rigid elements to the context of n-fold Pfister forms in two ways and study the relations between n-rigid and super n-rigid elements in various classes of fields including global fields, and in abstract Witt rings. In special cases the existence of higher rigidities turns out to be equivalent to important properties of fields known in literature. Among these are the property A_n, torsion freeness of I^nF and finite stability index of WF.

Introduction. Rigid elements have proved to be of importance in studying the structure of the Witt ring of a field (see, for instance, [**1**], [**2**], [**3**], [**4**], [**18**]). However, the notion of rigidity has strictly binary character and almost all existing approaches to rigid elements revolve around the defining condition $D_F\langle 1, x\rangle = \dot{F}^2 \cup x\dot{F}^2$. It is easy to find equivalent conditions which suggest the general notion of rigidity defined in terms of n-fold Pfister forms and powers of the fundamental ideal of Witt ring. Thus an element $x \in \dot{F}$ is said to be n-rigid if every n-fold Pfister form ϕ annihilated by $\langle 1, x\rangle$ in WF has to have the factor $\langle 1, -x\rangle$ making $\phi \cdot \langle 1, x\rangle = 0$ a triviality. Super n-rigidity is obtained by replacing Pfister forms with elements of I^nF. It turns out that what is easily seen to be equivalent for $n = 1$ is presumably a hard problem when $n > 2$. In this paper we make an attempt to understand what lies behind these general conditions.

In the first section we work in arbitrary fields (of characteristic not two). We find a characterization of n-rigid elements by value sets of quadratic forms and show that the sets of n-rigid elements form an ascending chain for $n = 1, 2, \ldots$. The same is proved for super n-rigid elements for special classes of fields. This leads to a new characterization of linked fields (Proposition 1.23). In special cases we arrive at interesting properties of fields studied in the literature. Thus n-rigidity of $x = 1$ means the field satisfies Elman and Lam's property A_n (cf. [**8**]) and super n-rigidity of $x = 1$ means I^nF is torsion free. And for $x = -1$ the two rigidity properties coincide with WF being $(n - 1)$-stable (cf. [**6**]). As a by-product we obtain a characterization of stability in terms of value groups of Pfister forms (see (1.11) below).