PACIFIC JOURNAL OF MATHEMATICS
Vol. 129, No. 1, 1987

MAXIMAL FUNCTIONS ON THE UNIT n-SPHERE

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It is shown that the Hardy-Littlewood maximal function on the unit sphere in n-space is weak-type (1, 1) with a weak-type constant cn where c is independent of n.

Introduction. E. M. Stein and J. O. Strömberg [6] have shown that the Hardy-Littlewood maximal function in $\mathbf{R}^n$ is weak-type $(1,1)$ with a weak-type constant cn with c independent of n. Their approach is to pointwise bound the maximal function by a supremum of averages of members of a certain heat-diffusion semi-group on $\mathbf{R}^n$. They then apply the Hopf abstract maximal ergodic theorem to obtain their result.

We plan to use an analogous version of this approach to show that the maximal function on the unit n-sphere is weak-type $(1,1)$ with a weak-type constant cn. The best weak-type constant prior to this was $cn\sqrt{n}$, see [4], using an entirely different approach.

Many of the ideas in this paper have already been presented in a paper by C. Herz [3]. In order to obtain the weak-type constant cn, sharper estimates are required than are indicated in Herz's paper. Furthermore, there is an oversight of a primarily technical nature which led this author to perform some contortions to rectify. It should be pointed out that Herz's overall approach applies not only to the unit sphere in $\mathbf{R}^n$ and $\mathbf{R}^n$ itself, but to more general spaces as well.

The author is appreciative of the informative comments and helpful suggestions of N. Stanton, E. M. Stein, and the referee.

NOTATION AND DEFINITIONS. Let S^{n-1} denote the unit sphere in $\mathbf{R}^n$ centered at the origin. Let ω_{n-1} denote its Lebesgue measure (surface area). Let $v(x,t) = T^t f(x)$ be the solution to the initial value problem $\partial v/\partial t = \Delta_S v$ and $v(x,0) = f(x)$ where Δ_S is the spherical Laplacian; that is, the "angular" part of the Laplacian in $\mathbf{R}^n$. If there is any confusion on the reader's part, Δ_S is defined precisely in the proof of Lemma 2 in equation (12).

Define the maximal heat function of f to be

$$M_T f(x) = \sup_{\lambda>0}\left|\frac{1}{\lambda}\int_0^\lambda T^\mu f(x)\,d\mu\right|.$$