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UNIQUENESS OF INFINITE DELOOPINGS FOR K-THEORETIC SPACES

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A functor Φ_p is constructed from spaces to spectra such that, for each spectrum X, $\Phi_p\Omega^\infty X$ is the p-adic completion of the K-theoretic localization of X. This functor is used to obtain uniqueness results for infinite deloopings of K-theoretic spaces and maps, thereby generalizing results of Adams-Priddy and Madsen-Snaith-Tornehave. Non-unique deloopings of K-theoretic maps are shown to involve phantom maps of spectra, and such maps are analyzed.

Introduction. Let K be the spectrum of nonconnective complex K-theory and recall that the associated homology theory K_* determines a localization functor $(-)_K$ on the homotopy category of spaces and of spectra by [**9**], [**10**], and [**12**]. In this paper we establish a natural equivalence $\Phi_p\Omega^\infty X \simeq (X_K)^\wedge_p$ for each prime p and spectrum X, where $\Omega^\infty X$ is the 0th space of the associated Ω-spectrum of X, where $(-)^\wedge_p$ is the p-adic completion functor, and where Φ_p is a new functor from spaces to p-adically complete K_*-local spectra. Thus $\Phi_p\Omega^\infty X \simeq X$ when X is a p-adically complete K_*-local spectrum and Ω^∞ therefore embeds the homotopy category of such spectra faithfully into the ordinary pointed homotopy of spaces.

In [**7**], Adams and Priddy showed by specific calculations that $BSO^\wedge_p$ and $BSU^\wedge_p$ have unique infinite deloopings, i.e., that there are unique homotopy types of connective spectra X and Y such that $\Omega^\infty X \simeq BSO^\wedge_p$ and $\Omega^\infty Y \simeq BSU^\wedge_p$. Using Φ_p we show that this uniqueness phenomenon occurs much more generally: for instance, if E is the $(n-1)$-connected section of a p-adically complete K_*-local spectrum, then the space $\Omega^\infty E$ has a unique infinite delooping when $n \geq 3$ or when $n = 2$ and $\pi_2 E$ is torsion. We obtain unique infinite deloopability results for p-adic completions of various infinite classical groups, their classifying spaces, their homogeneous spaces, and their J-spaces. We likewise generalize the p-local version of the Adams-Priddy theorem by proving unique infinite deloopability for localizations of these spaces at arbitrary finite sets of primes. We also generalize results of Madsen-Snaith-Tornehave [**19**] on