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MATRIX RINGS OVER *-REGULAR RINGS AND PSEUDO-RANK FUNCTIONS

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In this paper we obtain a characterization of those *-regular rings whose matrix rings are *-regular satisfying LP $\overset{*}{\sim}$ RP. This result allows us to obtain a structure theorem for the *-regular self-injective rings of type I which satisfy LP $\overset{*}{\sim}$ RP matricially.

Also, we are concerned with pseudo-rank functions and their corresponding metric completions. We show, amongst other things, that the LP $\overset{*}{\sim}$ RP axiom extends from a unit-regular *-regular ring to its completion with respect to a pseudo-rank function. Finally, we show that the property LP $\overset{*}{\sim}$ RP holds for some large classes of *-regular self-injective rings of type II.

All rings in this paper are associative with 1.

Let R be a ring with an involution *. Recall that * is said to be *n-positive definite* if $\sum_{i=1}^{n} x_i x_i^* = 0$ implies $x_1 = \cdots = x_n = 0$. The involution * is said to be *proper* if it is 1-positive definite; and if * is n-definite positive for all n, then we say that * is *positive definite*.

Recall than an element $e \in R$ is said to be a *projection* if $e^2 = e^* = e$ and R is called a *Rickart *-ring* if for every $x \in R$ there exists a projection e in R generating the right annihilator of x, that is $\imath(x) = eR$. Because of the involution, we have $\ell(x) = Rf$ for some projection f. Notice that $\imath(x) \cap x^*R = 0$, hence the involution * is proper and R is nonsingular. The above projections e, f depend on x only, $1 - e$ $(1 - f)$ is called the right (left) projection of x and, as usual, we shall write $1 - e = \mathrm{RP}(x)$, $1 - f = \mathrm{LP}(x)$.

If R is a *-ring, we denote by $\mathrm{P}(R)$ the set of projections of R partially ordered by $e \leq f$ iff $ef = e$. Thus, if $e \leq f$ we have $eR \subseteq fR$ and $Re \subseteq Rf$. Recall [**2**, pg. 14] that if R is Rickart, then $\mathrm{P}(R)$ is a lattice.

Two idempotents e, f of a ring R are said to be *equivalent*, $e \sim f$, if there exist $x \in eRf$, $y \in fRe$ such that $xy = e$, $yx = f$. If e, f are projections in a ring with involution and we can choose $y = x^*$ then e, f are said to be **-equivalent*, $e \overset{*}{\sim} f$. A ring is *directly finite* if $e \sim 1$ implies $e = 1$. A ring with involution is said to be *finite* if $e \overset{*}{\sim} 1$ implies $e = 1$.