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SCALE-INVARIANT MEASURABILITY IN ABSTRACT WIENER SPACES

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In this paper, we first prove a limit theorem for a sequence of quadratic functionals on an abstract Wiener space which generalizes a Cameron-Martin limit theorem in the Wiener space; and next we prove a version of a converse measurability theorem for the Wiener space in the setting of abstract Wiener spaces. Using these results, we discuss scale-invariant measurability and translations in an abstract Wiener space.

1. Introduction and preliminaries. Let H be a real separable infinite dimensional Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $|\cdot|$. Let μ be the cylinder set measure on H defined by

$$\mu(A) = (2\pi)^{-n/2} \int_F \exp\left\{-\frac{|x|^2}{2}\right\} dx,$$

where $A = P^{-1}(F)$, F is a Borel set in the image of an n-dimensional projection P in H and dx is Lebesgue measure in PH. A norm $\|\cdot\|$ on H is called *measurable* if for every $\varepsilon > 0$ there exists a finite dimensional projection P_0 such that $\mu(\{x \in H: \|Px\| > \varepsilon\}) < \varepsilon$ whenever P is a finite dimensional projection orthogonal to P_0. It is known (see [**8**]) that H is not complete with respect to $\|\cdot\|$. Let B denote the completion of H with respect to $\|\cdot\|$. Let i denote the natural injection from H into B. The adjoint operator i^* is one-to-one and maps B^* continuously onto a dense subset of H^*. By identifying H^* with H and B^* with i^*B^*, we have a triple $B^* \subset H \subset B$ and $\langle x, y \rangle = (x, y)$ for all x in H and y in B^*, where $(\cdot, \cdot)$ denote the natural dual pairing between B and B^*. By a well-known result of Gross, $\mu \circ i^{-1}$ has a unique countably additive extension ν to the Borel σ-algebra $\mathscr{B}(B)$ of B. The triple (H, B, ν) is called an *abstract Wiener space*. For more details, see Kuo [**8**].

Let $C_1[a, b]$ denote the Banach space $\{x(\cdot)$: x is a continuous function with $x(a) = 0\}$ with the uniform norm. Let $(C_1[a, b], \mathscr{B}(C_1[a, b]), m_w)$ denote the Wiener space, where m_w is the Wiener measure on the Borel σ-algebra $\mathscr{B}(C_1[a, b])$ of $C_1[a, b]$ (see [**12**]), and let $C_1'[a, b] = \{x \in C_1[a, b]: x(t) = \int_a^t f(u)\,du,\ f \in L^2[a, b]\}$; then it is a real separable infinite dimensional Hilbert space with the inner product $\langle x_1, x_2 \rangle = \int_a^b Dx_1(t) \cdot Dx_2(t)\,dt$, where $Dx = dx/dt$.