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A CHARACTERIZATION OF PSEUDO-ANOSOV FOLIATIONS

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Let M be a closed oriented smooth surface of genus $g \geq 2$, and let $\mathcal{MF}$ denote the space of equivalence classes of measured foliations on M. The importance of measured foliations began with Thurston's work on diffeomorphisms of surfaces: he defined the space $\mathcal{MF}$ and recognized the natural action of the mapping class group on $\mathcal{MF}$ as an extension of the action of this group on the Teichmüller space of M. In these investigations, there arose the concept of a pseudo-Anosov map which fixes a pair of transverse projective measured foliation classes on M, and the question evolves of recognizing the foliation classes fixed by some pseudo-Anosov map. Our main result provides a solution to this problem: we give a combinatorial characterization of these projective measured foliation classes. The combinatorial formulation of this problem uses the theory of train tracks.

1. Introduction.

1.1. In §§2 and 3 of this paper, we develop a method which associates a semi-infinite combinatorial "RLS word" to a class of measured foliations. The techniques underlying these RLS words first arose in [**K**] (see also [**HP**]) and depend on the machinery of train tracks; we recall the necessary material in §1.2. The RLS word does not uniquely determine the projective measured foliation class, rather it determines exactly the subset of $\mathcal{MF}$ consisting of the foliations topologically equivalent to the given foliation. (For the definitions and basic properties of measured foliations, we refer the reader to [**FLP**].) Section 4 contains our main results, and we completely characterize the classes of measured foliations left invariant by some pseudo-Anosov map in terms of their RLS words. Roughly, a measured foliation class is invariant under a pseudo-Anosov map if and only if it admits a preperiodic RLS word. Part of the theorem is in some sense constructive, and we describe how to find a pseudo-Anosov map fixing an invariant foliation class. This allows the description of an algorithm for producing representatives of all conjugacy classes of pseudo-Anosov map in §4.4, wherein we also discuss some open problems and likely applications.