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CURVATURE PROPERTIES OF TYPICAL CONVEX SURFACES

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Here we shall see that on typical convex surfaces the set of points with an infinite sectional curvature in some direction and that of points in which the lower sectional curvature in some direction equals the upper sectional curvature in the opposite direction are dense. Also we shall see that, in a certain sense, most convex surfaces are a.e. "very close" to their tangent hyperplane, closer than vanishing curvature already indicates.

Introduction. This paper completes the description of the curvature behaviour of most convex surfaces, the words "most" and "typical" being used in the sense of "all, except those in a set of first Baire category". It is known that typical convex surfaces are smooth and strictly convex (V. Klee [**7**]), but not of class C^2 (P. Gruber [**4**]). The latter result was strengthened by R. Schneider [**10**] and myself [**12**], [**13**]. Schneider proved that for these surfaces there is a dense set of points in which, for every tangent direction, the lower and upper curvatures are 0 and ∞ respectively. In [**12**] we showed that in each point where a finite curvature exists (and it exists almost everywhere by results of H. Busemann-W. Feller [**3**] and A. D. Aleksandrov [**1**]), the curvature is zero. In [**13**] we proved that the mentioned set in Schneider's result is not only dense, but also residual, i.e. a set of typical points.

For a survey on the use of Baire categories in Convexity, the reader may consult [**15**].

We consider the space $\mathscr{C}_n$ of all closed convex surfaces in $\mathbf{R}^n$. It is an easy matter to verify that $\mathscr{C}_n$, equipped with the Hausdorff distance, is a Baire space.

Let C be a smooth surface in $\mathscr{C}_n$, $x \in C$, τ be a tangent direction at x. We denote by $\rho_i^\tau(x)$ the lower radius of curvature at x in direction τ of the normal section of C (or of C itself for $n = 2$) along τ (see [**2**], p. 14 for a definition); analogously, the upper radius of curvature at x in direction τ is denoted by $\rho_s^\tau(x)$. For $n = 2$ there are at every point just two (opposite) tangent directions, which we simply denote by $+$ and $-$. If $\rho_i^\tau(x) = \rho_s^\tau(x)$, we write $\rho^\tau(x)$ for the common value. If $\rho^+(x) = \rho^-(x)$, we denote the common value by $\rho(x)$.