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WEAKLY COMPACT HOLOMORPHIC MAPPINGS ON BANACH SPACES

RAYMOND A. RYAN

A holomorphic mapping f: $E \to F$ of complex Banach spaces is weakly compact if every $x \in E$ has a neighbourhood V_x such that $f(V_x)$ is a relatively weakly compact subset of F. Several characterizations of weakly holomorphic mappings are given which are analogous to classical characterizations of weakly compact linear mappings and the Davis-Figiel-Johnson-Pełczynski factorization theorem is extended to weakly compact holomorphic mappings. It is shown that the complex Banach space E has the property that every holomorphic mapping from E into an arbitrary Banach space is weakly compact if and only if the space $\mathscr{H}(E)$ of holomorphic complex-valued functions on E, endowed with the bornological topology τ_δ, is reflexive.

1. Introduction. Aron and Schottenloher [3], in a study of the approximation property for locally convex spaces of holomorphic functions on a complex Banach space, introduced the concept of a compact holomorphic mapping. If E and F are complex Banach spaces, a holomorphic mapping f: $E \to F$ is said to be compact if every $x \in E$ has a neighbourhood V_x such that $f(V_x)$ is a relatively compact subset of F. They obtained several characterizations of compact holomorphic mappings which were analogous to characterizations of compact linear mappings. For example, it is well-known that a continuous linear mapping T: $E \to F$ is compact if and only if its transpose T^t: $F' \to E'$ is compact. Now consider a holomorphic mapping f: $E \to F$. Denoting by $\mathscr{H}(E)$ the vector space of holomorphic functions on E (that is, holomorphic mappings from E into **C**), the transpose of f may be defined as the linear mapping f^t: $F' \to \mathscr{H}(E)$ given by $f^t(\psi) = \psi \circ f$ for $\psi \in F'$. Aron and Schottenloher show that when $\mathscr{H}(E)$ is given a suitable locally convex topology, the holomorphic mapping f is compact if and only if its transpose f^t is a compact linear mapping of F' into $\mathscr{H}(E)$.

Motivated by this work, we carry out a similar study of weakly compact holomorphic mappings, which are defined in the same way: a holomorphic mapping f: $E \to F$ is weakly compact if every $x \in E$ has a neighbourhood V_x such that $f(V_x)$ is a relatively weakly compact subset of F. We extend some of the classical theory of weakly compact linear mappings to the holomorphic setting. Of course, some of our results are