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TRANSITIVE ISOMETRY GROUPS WITH NON-COMPACT ISOTROPY

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Let G be a connected Lie group acting effectively and transitively by isometries on a riemannian manifold M. Then G is a Lie subgroup of the full isometry group, which is not necessarily closed. In this paper we study the structure of the closure of G in $I(M)$ and illustrate the results with examples, with non-compact isotropy, where the closure is described explicitly.

Introduction. If M is a riemannian manifold the isotropy group in $I(M)$ is compact; hence a homogeneous riemannian M can always be represented as a quotient G'/H' with H' compact.

Assume now that G is a connected Lie group acting effectively and transitively by isometries on M. Then G is a Lie subgroup of $I(M)$ which will be closed in $I(M)$ if and only if the isotropy subgroup H is compact.

In this paper we study in detail the closure of G in $I(M)$. Also if G is any connected Lie group and H a closed subgroup we compare three standard conditions on H which ensure that G/H admits a riemannian invariant structure. The rest of the paper is devoted to illustrate the fact that it is rather common to have transitive, effective, non-closed Lie subgroups of $I(M)$, hence the isotropy subgroup is non-compact. This situation arises quite frequently, even when M is compact (Lemma 1.4). Also, any connected semisimple Lie group with infinite center admits a closed non-compact subgroup H such that G acts effectively on G/H and G/H carries a G-invariant riemannian structure. In this case, that is, when G is semisimple, we give an upper bound for the dimension of $\overline{G}_L$ and provide examples showing that these bounds are sharp (see (2.4), Proposition 2.3 and Remark 2.4).

In **[DMW]** the use of $\overline{G}_L$ proved to be convenient in the study of bounded isometries on a riemannian manifold acted on transitively and effectively by a semisimple Lie group without local compact factors. Also, some examples where G_L is not closed in $I(M)$ (G semisimple) are given in **[DMW]** Example (3.10). The authors would like to thank J. A. Wolf for very useful comments on a first version of this paper and, in particular, for suggesting a simpler proof of Proposition 2.2.