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WEIERSTRASS POINTS WITH TWO PRESCRIBED NON-GAPS

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In this paper, we study Weierstrass points P on smooth curves with two prescribed non-gaps n and s such that $s = en + d$ with $0 < d < n$. Let $\mathscr{M}$ be a fine moduli space of smooth curves of genus g (with some extra structure) and let $\not{p}: \mathscr{X} \to \mathscr{M}$ be the associated universal family. Let $\mathring{W}_{n,s} = \{x \in \mathscr{X}: n$ is the first non-gap of x and $\dim(|sx|) \geq e + 1\}$. Let Z be an irreducible component of $\mathring{W}_{n,s}$ and assume that $|sx|$ is a simple linear system on $\not{p}^{-1}(\not{p}(x))$ if x is a general point on Z. We prove that $\dim(Z) = n + s + g - 4 - e$ and $\dim(|sx|) = e + 1$. We give an example which shows that we cannot omit the assumption "$|sx|$ is a simple linear system". We prove that such a component Z exists if and only if $e(n-1) + d \leq g \leq ((n-1)(s-1) + 1 - (n,s))/2$. Finally, we derive some existence results of Weierstrass points.

Introduction. Let C be a smooth, irreducible curve of genus $g \geq 1$ defined over the field $\mathbf{C}$ of the complex numbers. Let p be a point on C. Let $n \in \mathbf{Z}_{\geq 1}$. We write $h^0(np)$ instead of $\dim(H^0(C, \mathscr{O}_C(np)))$.

We say that *n is a gap of p* if $h^0((n-1)p) = h^0(np)$. Otherwise, *n is a non-gap of p*.

Using the Theorem of Riemann-Roch, one can prove that the number of gaps of p is equal to g and each one of them is at most equal to $2g - 1$.

We say that *$(n_1, n_2, \ldots, n_g)$ is the gap sequence of p* if $1 \leq n_1 < n_2 < \cdots < n_g < 2g$ and n_i is a gap of p for each $1 \leq i \leq g$. We say that *p is a Weierstrass point of C* if $n_g \neq g$. One can prove that C has only a finite number of Weierstrass points. (For a detailed study of Weierstrass points see e.g. [**14**], §7d.)

Let $S \subset \mathbf{Z}_{\geq 0}$ be a sub-semigroup of $\mathbf{Z}_{\geq 0}$. Assume that there exists $c \in S$ such that $c + \mathbf{Z}_{\geq 0} \subset S$. Let $1 \leq n_1 < n_2 < \cdots < n_{g(S)}$ be such that $m \in Z_{\geq 0} \setminus S$ if and only if $m = n_i$ for some $1 \leq i \leq g(S)$. We call $(n_1, \ldots, n_{g(S)})$ a *gap sequence of genus $g(S)$*. If there exists a smooth curve C (of genus $g(S)$) and a point p on C such that $(n_1, n_2, \ldots, n_{g(S)})$ is the gap sequence of p, then we say that $(n_1, n_2, \ldots, n_{g(S)})$ is a *Weierstrass gap sequence*. It is an open problem in the theory of Weierstrass points to decide which gap sequences are Weierstrass gap sequences. There exist gap sequences which are not Weierstrass gap sequences (see [**4**]).