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TOPOLOGICAL IDENTIFICATION OF MULTIPLE SOLUTIONS TO PARAMETRIZED NONLINEAR EQUATIONS

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Dedicated to the memory of James Dugundji

Let $L: E \to F$ be an isomorphism of Banach spaces, let $H: E \times \mathbf{R}^n \to F$ be a completely continuous mapping, and let $B: E \to \mathbf{R}^n$ be a bounded linear mapping onto a euclidean space. The solutions (y, λ) to the problem

$$(*) \qquad \begin{cases} Ly = H(y, \lambda), \\ By = 0 \end{cases}$$

can be represented as the fixed points of a mapping $T: E \times \mathbf{R}^n \to E \times \mathbf{R}^n$. Neilsen fixed point theory may be extended to produce lower bounds for the number of fixed points of such maps. Problems of the type $(*)$ include boundary value problems for ordinary differential systems of the form:

$$\begin{cases} y'' = h(x, y, y', \lambda), \\ y(0) = y(1) = 0, \end{cases}$$

where $y = y(x): [0,1] \to \mathbf{R}^n$ and $\lambda \in \mathbf{R}^n$, satisfying an additional condition such as $y(1/2) = 0$ or $\int_0^1 y(t)\,dt = A$ for a given $A \in \mathbf{R}^n$.

A standard technique in nonlinear analysis, for establishing that an equation has a solution, consists of finding a mapping whose fixed points correspond to the solutions of the equation and then applying a topological fixed point theorem. A fixed point theory initiated by Jakob Nielsen in the 1920s is concerned with the number (rather than just the existence) of fixed points [**8**]. In 1950, Jean Leray [**11**] suggested that Nielsen fixed point theory might therefore be useful in proving that equations have multiple solutions. In this paper, we apply Neilsen fixed point theory to a class of parametrized equations in which the parameter space is finite-dimensional.

Section 1 is concerned with the modification of some techniques from [**1**], to obtain a form of Nielsen theory suited to the analytic problem. Section 2 presents a description of the problem in operator theoretic language and contains the main result of the paper (Theorem 2.3) which gives sufficient conditions for the topological methods of the first section