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THE ORIENTED HOMOTOPY TYPE OF SPUN 3-MANIFOLDS

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We show that, bar unexpected developments in 3-manifold theory, the fundamental group and the choice of framing determine the oriented homotopy type of spun 3-manifolds.

1. The object of this note is to classify spun 3-manifolds up to oriented homotopy type. The notion of spinning was introduced by Artin [**1**] in the context of knots. The asphericity of classical knots implies that spun knots with isomorphic fundamental groups have homotopy equivalent complements. What we do is extend this to closed manifolds.

Let M^3 be a closed, oriented 3-manifold, and $\mathring{M}$ be M with an open 3-ball removed. Gordon [**4**] defines the *spin* of M to be the closed, oriented, smooth 4-manifold $s(M) = \partial(\mathring{M} \times D^2)$. Note that $s(M)$ is obtained by gluing $\mathring{M} \times S^1$ to $S^2 \times D^2$ via $\mathrm{id}_{S^2 \times S^1}$. There is one other possible choice of gluing map, the "Gluck twist" τ: $((\theta, \phi), \psi) \mapsto ((\theta + \psi, \phi), \psi)$ corresponding to $\pi_1(\mathrm{SO}(3)) \cong \mathbf{Z}_2$. The resulting manifold $s'(M) = \mathring{M} \times S^1 \cup_\tau S^2 \times D^2$ is called the *twisted spin* of M [**9**]. The two spins of M have the same fundamental group as M. In fact, they have identical 3-skeleta, but different attaching maps for the 4-cell. If M admits a circle action with fixed points (e.g. M is a lens space), then $s(M) \cong s'(M)$, but if M is aspherical $s(M) \not\simeq s'(M)$, as shown by Plotnick [**11**].

Every closed, oriented M^3 admits a (unique up to order) connected sum decomposition $M_1 \# M_2 \# \cdots \# M_n$, with prime factors M_i either aspherical, spherical, or $S^2 \times S^1$ (see e.g. [**6**]). The spherical factors are of the form Σ^3/π, with Σ^3 a homotopy 3-sphere and π a finite group acting freely on Σ^3. Consider only manifolds M^3 satisfying the condition

(1.1) All spherical factors are either homotopy 3-spheres or spherical Clifford-Klein manifolds (i.e. S^3/π, π acting linearly).

Under this assumption (no counterexamples are known!), we will prove the following

THEOREM 1.2. *If $\pi_1(M) \cong \pi_1(M')$, then $s(M) \simeq s(M')$ and $s'(M) \simeq s'(M')$.*