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CONVERGENCE FOR THE SQUARE ROOT OF THE POISSON KERNEL

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Let X be a symmetric space and f an integrable function on its boundary ∂X. The 0-Poisson integral $P_0 f$ is the function on X obtained by integrating f against the square root of the Poisson kernel. We give Fatou theorems saying that the normalized function $P_0 f/P_0 1$ converges almost everywhere to f on ∂X. Many such results are known for λ-Poisson integrals $P_\lambda f$ with λ in the positive Weyl chamber. But the case $\lambda = 0$ is different, since larger regions of convergence can be used. Some of our results are general, some are given for the bidisk or $SL(3, \mathbf{R})/SO(3)$. The paper extends previous results by the author for the disk and the bidisk.

1. Introduction. Let $P = P(z, e^{i\theta})$ denote the Poisson kernel in the unit disk $\mathbf{D}$. If f is an L^1 function on $\mathbf{T} = \partial\mathbf{D}$ and $\lambda \in \mathbf{R}$, the λ-Poisson integral of f is

$$P_\lambda f(z) = \int_{\mathbf{T}} P(z, e^{i\theta})^{\lambda+1/2} f(e^{i\theta})\, d\theta.$$

It is an eigenfunction of the hyperbolic Laplacian in $\mathbf{D}$, with eigenvalue $4\lambda^2 - 1$. The corresponding normalized λ-Poisson integral is $\mathscr{P}_\lambda f = P_\lambda f/P_\lambda 1$, where 1 is the constant function on $\mathbf{T}$. Assume $\lambda \geq 0$. Then $\mathscr{P}_\lambda f(z) \to f(e^{i\theta})$ as $z \to e^{i\theta}$, uniformly for continuous f. This is not true when $\lambda < 0$.

Let $f \in L^1$. For $\lambda > 0$ it is known that $\mathscr{P}_\lambda f$ tends nontangentially to f a.e. in $\mathbf{T}$. This means that one lets $z \to e^{i\theta}$ satisfying a condition $|\arg z - \theta| < \text{const.}(1 - |z|)$. But when $\lambda = 0$, more is true. In fact, $\mathscr{P}_0 f(z) \to f(e^{i\theta})$ as $z \to e^{i\theta}$, $|\arg z - \theta| < \text{const.}\,(1 - |z|)\log(1 - |z|)^{-1}$ for a.a. θ, see Sjögren [**16**]. We call this weakly tangential convergence; it is false for $\mathscr{P}_\lambda f$, $\lambda > 0$. J. Taylor has found a simple description of it in terms of the hyperbolic metric.

In this paper, we shall give some convergence results for $\mathscr{P}_0 f$ in symmetric spaces, most of which are not valid for other $\mathscr{P}_\lambda f$. For rank 1 spaces, the generalization of weakly tangential convergence is rather straightforward, see Theorem 5.4 below. A metric description of the approach region appearing here is also given. Korányi and Picardello [**11**]