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JONES POLYNOMIALS OF PERIODIC LINKS

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Let L be a link in S^3 which has a prime period and L_* be its factor link. Several relationships between the Jones polynomials of L and L_* are proved. As an application, it is shown that some knot cannot have a certain period.

1. Introduction. Let L be an oriented link that has period $r > 1$. That is, there exists an orientation preserving auto-homeomorphism $\phi: S^3 \to S^3$ of order r with a set of fixed points $F \cong S^1$ disjoint from L and which maps L onto itself. By the positive solution of Smith Conjecture, F is unknotted. Let $\Sigma^3 = S^3/\phi$ be the quotient space under ϕ. Since F is unknotted, Σ^3 is again a 3-sphere, and S^3 is the r-fold cyclic covering space of Σ^3 branched along F.

Let $\psi: S^3 \to \Sigma^3$ be the covering projection. Denote $\psi(L) = L_*$, which is called the *factor link*, and let $V_L(t)$ and $V_{L_*}(t)$ denote, respectively, the Jones polynomials of L and L_*.

In this paper, we will prove some relationships between $V_L(t)$ and $V_{L_*}(t)$ which are analogous to those between their Alexander polynomials [**M2**]. In fact, we will prove

THEOREM 1. *Let r be a prime and L a link that has period r^q, $q \geq 1$. Then*

$$V_L(t) \equiv \left[V_{L_*}(t)\right]^{r^q} \mod(r, \xi_r(t)), \tag{1.1}$$

where $\xi_r(t) = \sum_{j=0}^{r-1}(-t)^j - t^{(r-1)/2}$.

If L is not split, then we are able to prove a slightly more precise formula.

Let $\operatorname{lk}(X, Y)$ denote the linking number between two simple closed curves X and Y in S^3. Then we have

THEOREM 2. *Let r be a prime and L a non-split link that has period r^q, $q \geq 1$.*