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MODULES SATISFYING ACC ON A CERTAIN TYPE OF COLONS

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Let M be a module over a ring R, which satisfies the ascending chain condition on submodules of the form $N: B \subseteq N: B^2 \subseteq N: B^3 \subseteq \cdots$ for every submodule N of M and every finitely generated ideal B of R. We investigate the class of such modules M and show that various important properties of Noetherian modules and rings can be generalized to modules and rings of this class.

Introduction. Let M be a module over a ring R (commutative with identity). M is said to satisfy (accr) (resp. (accr*)) if the ascending chain of residuals of the form $N: B \subseteq N: B^2 \subseteq N: B^3 \subseteq \cdots$ terminates for every submodule N of M and every finitely generated (resp. principal) ideal B of R. The class of modules satisfying (accr) is large. It contains Noetherian modules, modules over Artinian rings, modules having ACC on colon submodules ([**14**]), Laskerian modules, modules over perfect rings, etc. The purpose of this paper is to investigate this class of modules and show that these modules enjoy various important properties of Noetherian modules.

In §1, we prove that (accr) and (accr*) are equivalent properties of modules. We also characterize modules satisfying this property as those R-modules M in which every submodule N can be written in the form $N = (N: B^h) \cap (N + B^hM)$ for every finitely generated ideal B of R and for all sufficiently large positive integers h.

Section 2 deals with fundamental properties of modules satisfying (accr). The main result is that both a weak version of the Artin-Rees Lemma and the Krull Intersection Theorem for Noetherian modules can be generated to modules with (accr). We also give an example of a module in order to show that every Laskerian module satisfies (accr), but a module satisfying (accr) is not necessarily Laskerian.

In §3, we consider conditions under which modules (resp. quasi-semi-local rings) satisfying (accr) are Laskerian (resp. Noetherian). Let M be a module each of whose factor module is finite dimensional [**10**]. We prove that such an R-module M is Laskerian $\Leftrightarrow$ M satisfies (accr) $\Leftrightarrow$ the Artin-Rees property holds for every submodule N of M and every principal ideal (r) of R, that is $N \cap r^hM \subseteq rN$ for all sufficiently large integers h.