PACIFIC JOURNAL OF MATHEMATICS
Vol. 131, No. 2, 1988

A STUDY OF REGULARITY PROBLEM OF HARMONIC MAPS

GUOJUN LIAO

In this paper we study the regularity problem of harmonic maps between closed compact manifolds (M^n, g) and (N^m, h) in dimensions $n \geq 3$.

1. Introduction. Harmonic maps are critical points of the energy functional. For technical convenience we assume, by virtue of the Nash imbedding theorem, that the target manifold N is isometrically imbedded in the smallest Euclidean space $\mathbf{R}^k$. At the end of this section we will discuss the independence of our definitions on the imbedding of N in $\mathbf{R}^k$.

DEFINITION (1.1). A map $u = (u^1, u^2, \ldots, u^k)$: $M \to \mathbf{R}^k$ is said to belong to the Sobolev space $L_1^2(M, \mathbf{R}^k)$ if for $i = 1, 2, \ldots, k$

$$\int_M |\nabla u^i|^2 \, dV < \infty$$

where $|\nabla u^i|$ is the covariant derivative, in local coordinates,

$$|\nabla u^i|^2(x) = g^{\alpha\beta}(x) \cdot \partial_\alpha u^i \cdot \partial_\beta u^i,$$

dV is the volume element of M. For $u \in L_1^2(M, \mathbf{R}^k)$ one defines its energy as

$$E(u) = \sum_{i=1}^k \int_M |\nabla u^i|^2 \, dV = \int_M |\nabla u|^2 \, dV.$$

DEFINITION (1.2). A map u is said to belong to $L_1^2(M, N)$ if $u \in L_1^2(M, \mathbf{R}^k)$ and if $u(x) \in N$, a.e. $x \in M$.

REMARK. $L_1^2(M, \mathbf{R}^k)$ with the usual norm

$$|u|_{1,2} = \left(E(u) + \sum_{i=1}^k \int_M |u^i|^2 \, dV \right)$$

is a separable Hilbert space. $L_1^2(M, N)$ has strong and weak topologies induced from that of $L_1^2(M, \mathbf{R}^k)$. Moreover, the set

$$\{ u \in L_1^2(M, N) : |u|_{1,2} \leq C \}$$

is weakly compact in $L_1^2(M, \mathbf{R}^k)$.