PACIFIC JOURNAL OF MATHEMATICS
Vol. 131, No. 2, 1988

# WEIGHTED NORM INEQUALITIES FOR THE FOURIER TRANSFORM ON CONNECTED LOCALLY COMPACT GROUPS

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**Let $G$ be a locally compact connected group. If $G$ is also either compact or abelian, sufficient conditions on a non-negative pair of measurable functions $T$ and $V$ are given to imply that there exists a constant $c$ independent of $f$ for which an inequality of the form**

$$\left(\int_\Gamma |\hat{f}(\gamma)|^{p'} T(\gamma)\, dm_\Gamma(\gamma)\right)^{1/p'} \leq c\left(\int_G |f(x)|^p V(x)\, dm_G(x)\right)^{1/p}$$

**holds for all integrable $f$ on $G$ $(1 < p \leq 2, p' = p/(p-1))$. Here, $\hat{f}$ denotes the Fourier transform of $f$ defined on $\Gamma$ (the dual of $G$) and $m_G$, $m_\Gamma$ are Haar measures on $G, \Gamma$ respectively. Conditions on $T, V$ are also given to esnure that the inequality holds with $p'$ replaced by a more general exponent on a more restricted class of groups.**

**1. Introduction.** In [**6**], sufficient conditions are placed on non-negative pairs of functions $T, V$ to imply

$$\left(\int_{\mathbf{R}^n} |\hat{f}(x)|^q T(x)\, dx\right)^{1/q} \leq c\left(\int_{\mathbf{R}^n} |f(x)|^p V(x)\, dx\right)^{1/p}$$

for all integrable functions $f$ on $\mathbf{R}^n$, with $c$ a constant independent of $f$. Our aims here are to first make sense of such inequalities in a more abstract setting (in particular, for functions defined on a fairly broad class of locally compact groups $G$ and their Fourier transforms defined on the suitable dual object $\Gamma$ with $m_G$, $m_\Gamma$ being the appropriate measures on $G$ and $\Gamma$ respectively), and then to produce conditions on $T, V$ analogous to those found in [**6**] so that the inequality will be valid. In the case $1 < p \leq 2$ and $q = p' = p/(p-1)$, the main technical difficulty is the proof of the existence of a measurable function $W: G \to \mathbf{R}^+$ $(= [0, \infty))$ with the properties

$$\begin{aligned} &V(x) \leq 2W(x) < 2V(x) && \text{for all } x \in G \quad \text{and} \\ &m_G\{x \in G; W(x) = \alpha\} = 0 && \text{for all } \alpha > 0. \end{aligned} \tag{1.1}$$

In the case $q \neq p'$ however, it is necessary to have a measurable function $S: \Gamma \to \mathbf{R}^+$ satisfying

$$\begin{aligned} &T(\gamma) \leq 2S(\gamma) < 2T(\gamma) && \text{for all } \gamma \in \Gamma \quad \text{and} \\ &m_\Gamma\{\gamma \in \Gamma: S(\gamma) = \alpha\} = 0 && \text{for all } \alpha > 0. \end{aligned} \tag{1.2}$$