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MENGER SPACES AND INVERSE LIMITS

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M. Bestvina in 1984 characterized the Menger universal n-dimensional spaces. This characterization is used to identify certain inverse sequences having inverse limit homeomorphic to one of the Menger spaces. Specific models of the Menger spaces are then constructed in the Hilbert Cube as inverse limits of polyhedra. The union of these models is shown to be homeomorphic to the countably infinite dimensional space σ.

1. Introduction. In 1984, M. Bestvina [**Be**] characterized the Menger universal n-dimensional compactum μ_n as follows.

THEOREM. *A space X is homeomorphic to μ_n if and only if X satisfies the following properties*:

1. *X is compact and n-dimensional*,
2. *X is $(n-1)$-connected (C^{n-1})*,
3. *X is locally $(n-1)$-connected (LC^{n-1}), and*
4. *X satisfies the disjoint n-cells property (DD^nP).*

Using this characterization, Bestvina showed that the various constructions in the literature of compact universal n-dimensional spaces ([**Mg**], [**Lf**], [**Pa**]) all yield μ_n. In addition, Bestvina showed that each μ_n is homogeneous. Prior to this result, there had been characterizations only of μ_0 (the Cantor set) and μ_1 (the universal curve) [**An**].

Using Bestvina's characterization, we identify certain inverse sequences that have μ_n as inverse limit. This leads to the construction of models of μ_n in the Hilbert Cube. These models can be described by putting restrictions on the coordinates of points in the Hilbert Cube. We also show that the union of certain of these models naturally yields the countably infinite dimensional space σ.

2. Notation and terminology. All spaces are assumed to be separable and metrizable. A reference for dimension theory is [**En**]. A space is $n-1$ *connected*, C^{n-1}, if each map of S^k, $k \leq n-1$, into the space extends to a map of the $k+1$ cell into the space. A space X is *locally* $n-1$ *connected*, LC^{n-1}, if for each point $p \in X$, and for each neighborhood U of p there is a neighborhood V of p, $V \subset U$, so that each map of S^k into V, $k \leq n-1$, extends to a map of B^{k+1} into U.