PACIFIC JOURNAL OF MATHEMATICS
Vol. 131, No. 2, 1988

SEPARATION PROPERTIES AND EXACT RADON-NIKODYM DERIVATIVES FOR BOUNDED FINITELY ADDITIVE MEASURES

WAYNE C. BELL AND JOHN W. HAGOOD

Necessary and sufficient conditions for a μ-continuous, bounded finitely additive measure to have an exact Radon-Nikodym derivative are obtained in terms of a new separation property intermediate between disjointness and mutual singularity.

1. Introduction. In the classical Radon-Nikodym theorem, it is proved that for countably additive real valued measures on a σ-field of sets, the class of measures that are absolutely continuous with respect to a fixed measure μ is precisely the set of measures having a representation of the form $\int f\,d\mu$ for a μ-integrable function f. For bounded finitely additive measures on a field, these sets do not in general coincide. In this setting Radon-Nikodym theorems can have one of two goals: to characterize the absolute continuity class of μ or to provide necessary and sufficient conditions for a measure to have an exact Radon-Nikodym derivative with respect to μ. In the history of the Radon-Nikodym theorem for bounded finitely additive scalar measures, the characterization of absolute continuity was first to receive attention. Bochner [3] proved that in order for ν to be absolutely continuous with respect to μ, it is necessary and sufficient that ν be the limit in variation norm of a sequence of integrals of μ-simple functions. Variations of Bochner's theorem, all providing characterizations of absolute continuity in terms of limits of sequences of integrals, have been obtained for a variety of settings by Darst and Green [5], Fefferman [7], and Luxemburg [9], among others. However, none of these provide an exact Radon-Nikodym derivative.

The first successful characterization of those bounded finitely additive real valued measures that have an exact Radon-Nikodym derivative with respect to μ was given by Maynard [10]. In addition to absolute continuity, the necessary and sufficient conditions he obtained require certain intricate behavior of the average range function $A_\nu(E)$ on the field, where $A_\nu(E)$ is the set of ratios $\nu(F)/\mu(F)$ for $F \subseteq E$, $\mu(F) \neq 0$, which involves exhaustions of μ by related classes of sets. Maynard was able to simplify his conditions for those μ which admit an exhaustive Hahn