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THE CAMPBELL-HAUSDORFF GROUP AND A POLAR DECOMPOSITION OF GRADED ALGEBRA AUTOMORPHISMS

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Let $A = \prod_{k=k_0}^{\infty} \mathrm{gr}_k(A)$ be a complete graded (associative or Lie) algebra over a field of characteristic zero, filtered by the decreasing filtration $F_j(A) = \prod_{k=j}^{\infty} \mathrm{gr}_k(A)$. We let $\mathrm{Aut}(A)$ denote the group of filtration preserving automorphisms of A, and $\mathrm{Aut}_0(A)$ the subgroup consisting of those elements of $\mathrm{Aut}(A)$ which preserve the grading. In this paper we prove that every element of $\mathrm{Aut}(A)$ has a unique polar decomposition of the form $u_0 \exp(d)$, where $u_0 \in \mathrm{Aut}_0(A)$ and $d: A \to A$ is a filtration increasing derivation.

Our central results, presented in §4, generalize and were inspired by theorems on decompositions of diffeomorphisms and symplectic mappings found in the dynamical systems literature; they also touch on the related topic of one-parameter group extensions. Particularly influential were Broer's treatment of normal forms of vector fields [**4**], and Sternberg's work on the formal aspects of dynamical systems [**11**]. The setting adopted here is that of filtered groups and algebras, for the reason that certain functorial properties of these structures are particularly well suited for the treatment of convergence questions arising from the use of the Campbell-Hausdorff formula. Broer (loc. cit.) credits Gérard and Levelt [**6**] with the first use of filtration techniques in this field.

A second aspect of our work is the introduction of a restricted class of "analytic functions" which map the ground field into appropriate filtered objects. Such functions turn out to be rather peculiar, in that they are always "entire" and have (except when identically zero) only finitely many zeros. Our study is limited to those properties which are relevant to this paper.

Applications of Theorem 4.5 are presented in the last two sections; we offer short proofs of two of these decompositions. That implied by the upper exact sequence of Theorem 5.4 is classical: C. L. Bouton was working on related problems as early as 1916 [**2**] (also see Lewis [**8**] and Sternberg [**11**]). The decomposition implied by Theorem 6.2 is also well-known (see van der Meer [**14**, Lemma 2.11, p. 27]). The novelty of the