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SUMS OF PRODUCTS OF POWERS OF GIVEN PRIME NUMBERS

R. TIJDEMAN AND LIANXIANG WANG

We give the complete solutions of the equations $2^x3^y + 1 = 2^z + 3^w$, $2^x3^y + 2^z = 3^w + 1$ and $2^x3^y + 3^w = 2^z + 1$ in integers x, y, z, w. We use this to prove that every large rational number has at most four representations of the form $2^\alpha 3^\beta + 2^\gamma + 3^\delta$. Finally we prove that, for given integer n and prime numbers $p_1, \dots, p_t$, every rational number m has at most C representations of the form $\sum_{i=1}^n p_1^{k_{i1}} \cdots p_t^{k_{it}}$ where $k_{i1}, \dots, k_{it}$ are integers.

0. D. J. Newman conjectured that if $w(n)$ denotes the number of solutions of $n = 2^a + 3^b + 2^c3^d$ then $w(n)$ is bounded (see Erdős and Graham **[4]** p. 80). Evertse, Győry, Stewart and Tijdeman **[6]** Theorem 6(a) settled this conjecture. We call two representations $x_1 + \cdots + x_n$ and $x'_1 + \cdots + x'_n$ *distinct*, if the unordered tuples $(x_1, \dots, x_n)$ and $(x'_1, \dots, x'_n)$ are not the same. In §2 we prove that the number of distinct representations of a rational number m as $2^\alpha 3^\beta + 2^\gamma + 3^\delta$ is at most four, if m exceeds a certain constant. The number four is the best possible.

To prove this result we need not only the Main Theorem on S-Unit Equations (Lemma 4) as in **[6]**, but also the complete solutions of the diophantine equations mentioned in the first paragraph of this paper. Here we recall the remark of Brenner and Foster (**[2]** Comment 8.037) that the class of equations

$$1 + (pq)^a = p^b + q^c$$

where p, q are given distinct primes, does not seem to be amenable by their (congruential) method. We show in §1 how the more general equation

$$1 + p^xq^y = p^z + q^w$$

can be treated by Baker's method for estimating linear forms in the logarithms of algebraic numbers. The essential tool in §1 is Lemma 1, due to Ellison **[3]** and specially made for the primes $p = 2$, $q = 3$. De Weger **[10]** has proved a corresponding result for all primes p, q with $2 \le p < q \le 200$ and the method works for any pair of prime