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SEMIGROUPS GENERATED BY CERTAIN OPERATORS ON VARIETIES OF COMPLETELY REGULAR SEMIGROUPS

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The operators C, K, L, T, T_l and T_r on the lattice $\mathscr{L}(\mathscr{CR})$ of varieties of completely regular semigroups have played an important role in recent studies of $\mathscr{L}(\mathscr{CR})$. Although each of these operators is idempotent, when applied in various combinations to the trivial variety they yield varieties for which the only upper bound is $\mathscr{CR}$. The semigroups generated by various subsets of $\{C, K, L, T, T_r, T_l\}$ are determined here in terms of generators and relations.

1. Introduction and summary. Completely regular semigroups (unions of groups) may be regarded as algebras with the operations of (binary) multiplication and (unary) inversion. As such they form a variety $\mathscr{CR}$ defined by the identities

$$(1) \qquad (ab)c = (ab)c, \quad a = aa^{-1}a, \quad aa^{-1} = a^{-1}a, \quad (a^{-1})^{-1} = a.$$

The lattice $\mathscr{L}(\mathscr{CR})$ of all subvarieties of $\mathscr{CR}$ turns out to be amenable to a thorough analysis both globally and locally. The former includes various (complete) congruences that emerge naturally in the study either of the varieties themselves or of the corresponding fully invariant congruences on a free completely regular semigroup $F\mathscr{CR}$ on a countably infinite set. Local studies of the lattice $\mathscr{L}(\mathscr{CR})$ usually amount to rather complete descriptions of relatively small intervals in $\mathscr{L}(\mathscr{CR})$ modulo $\mathscr{L}(\mathscr{G})$, the lattice of group varieties, starting from the bottom of the lattice.

In the local approach, a number of operators make their appearance in the description of certain varieties in terms of some of their proper subvarieties. But these operators may be defined on all of $\mathscr{L}(\mathscr{CR})$ thereby providing a certain amount of information for varieties scattered throughout $\mathscr{L}(\mathscr{CR})$ and hence may be used for a global study of this lattice. Another source of operators on $\mathscr{L}(\mathscr{CR})$ are the kernel and trace relations on the lattice of fully invariant congruences on $F\mathscr{CR}$ now translated into relations on $\mathscr{L}(\mathscr{CR})$.

Of the considerable literature on varieties of completely regular semigroups, we mention only the following ones because they are directly related to our object of study. We thus cite Jones [6], [7], Kaďourek [8],