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THE NORMAL INDEX OF A FINITE GROUP

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For a maximal subgroup M of a finite group G the normal index of M is the order of a chief factor H/K where H is minimal in the set of supplements of M in G. We obtain results about the normal index of M when M has composite index in G.

1. Introduction. The relationships between the properties of maximal subgroups of a finite group G and the structure of G have been studied by many people. In [3], [4] and [10] we investigated maximal subgroups of composite index, developing analogs of the Frattini subgroup and studying their role in the structure of groups. Here we obtain results which involve the normal index (introduced by Deskins in [5]) of a maximal subgroup M of a group G. The normal index of M, $\eta(G:M)$, is the order of a chief factor H/K of G when H is a minimal supplement of M in G. In §§2–4 we obtain extensions of results of Deskins [5], Beidleman and Spencer [2] and Mukherjee [9] based on $\eta(G:M)$ for the case when $[G:M]$ is composite.

All groups treated are finite, notation is standard (from [6] and [8]), and a maximal subgroup M of G is often denoted by $M <\cdot G$. If $M <\cdot G$ and $[G:M]$ is composite we call M *c-maximal* in G.

2. Normal index and solvability. If M is a maximal subgroup of a group G and H is a minimal normal supplement to M in G then for any chief factor H/K of G it follows that $K \subseteq M$ and $G = MH$. Therefore we have that $[G:M]$ divides $o(H/K) = \eta(G:M)$. For the sake of completeness we first describe some properties of the normal index which we shall use subsequently.

2.1 (Deskins [5, 2.1], Beidleman and Spencer [2, Lemma 1]). If M is a maximal subgroup of a group G then $\eta(G:M)$ is uniquely determined by M.

2.2 (Beidleman and Spencer [2, Lemma 2]). If N is a normal subgroup of a group G and M is a maximal subgroup of G such that $N \subseteq M$ then $\eta(G/N : M/N) = \eta(G:M)$.