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ON THE BIRMAN INVARIANTS OF HEEGAARD SPLITTINGS

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Dedicated to Professor Joan S. Birman

As Professor Birman indicated in [Bi1] the homological information about a given Heegaard splitting of genus g is contained in a double coset in the group of symplectic $2g \times 2g$ integer matrices with respect to a suitable subgroup. She found in [Bi1] a determinant invariant of this double coset and we prove in this paper that that invariant (strengthened a bit when the first torsion number is even) is complete. We obtain this result by characterizing the double coset in terms of the linking form of the manifold lifted to a handlebody of the Heegaard splitting and by finding complete invariants of this lifted form. Professor Birman has kindly pointed out to us that the characterization of the double cosets by means of her invariant is contained in the unpublished manuscript [Bi-J].

1. Introduction. Let F be an orientable, closed surface embedded in a (closed, orientable) 3-manifold M. The pair (M, F) is said to be a *Heegaard splitting* if F separates M in two handlebodies. If the genus of F is g, then (M, F) is a Heegaard splitting of *genus* g. Two Heegaard splittings (M, F) and (M', F'), are *equivalent* if $(M, F) \cong (M', F')$, i.e. if there exists a homeomorphism $h\colon M \to M'$ such that $h(F) = F'$.

Every closed, orientable 3-manifold has Heegaard splittings as was remarked by Heegaard. A considerable effort has been made in the past to achieve the classification of Heegaard splittings. Waldhausen [**Wa**] proved that any two genus g Heegaard splittings of the 3-sphere S^3 are equivalent (ambient isotopic, indeed). He used the classical result of Reidemester and Singer that two Heegaard splittings of the same manifold are *stably* equivalent, i.e. they are ambient isotopic after adding enough trivial handles to both Heegaard splittings. Bonahon-Otal [**Bo-O**] classified the Heegaard splittings of the lens spaces and they showed that there is exactly one for each genus. This is not true in general: Engmann [**Eng**] gave the first example of a manifold (a connected sum of lenses) with two inequivalent Heegaard splittings of genus 2 (for a different proof and generalization see [**Bi1**]). Later Birman-González-Montesinos [**BGAM**] distinguished two Heegaard splittings of genus two of an irreducible manifold (a homology sphere), using geometric methods introduced earlier