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SIMPLICITY OF PARTIAL AND SCHMIDT DIFFERENTIAL OPERATOR RINGS

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In this paper we develop necessary and sufficient conditions for certain differential operator rings to be simple. We do this for a ring with finitely many commuting derivations and for a ring with a commuting Schmidt higher derivation of finite length. Also we give a correspondence between finite sets of derivations and Schmidt higher derivations.

In this first section we deal with the simplicity of a differential operator ring over a ring with finitely many commuting derivations. Many of the results of this section overlap those of others, among them Jordan [**8**], Hauger [**5**] and Voskoglou [**11**]. The second section deals with the construction of a Schmidt higher derivation from a set of derivations, and vice versa. The correspondence is the same as that of Heerema [**6**] but we give a different construction. The third section deals with simplicity of a differential operator ring over a ring with a Schmidt higher derivation. All rings are considered to be associative with unit. This research will form part of the author's Ph.D. dissertation at the University of Utah. The author wishes to thank K. R. Goodearl for his help and suggestions.

1. Let $\delta_1, \ldots, \delta_k$ be derivations on a ring R. A $(\delta_1, \ldots, \delta_k)$-*ideal* of R is any ideal I of R such that $\delta_i(I) \subseteq I$ for all i. The ring R is said to be $(\delta_1, \ldots, \delta_k)$-simple if R is nonzero and the only $(\delta_1, \ldots, \delta_k)$-ideals of R are 0 and R. The elements $r \in R$ such that $\delta_i(r) = 0$ for all i are called $(\delta_1, \ldots, \delta_k)$-*constants* and form a subring of R.

If $\delta_1, \ldots, \delta_k$ are commuting derivations on R, the formal linear differential operator ring

$$T = R[\Theta_1, \ldots, \Theta_k; \delta_1, \ldots, \delta_k]$$

is the free left R-module generated by the symbols

$$\Theta_k^{n(k)} \cdots \Theta_1^{n(1)}$$