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RINGS WHOSE KERNEL FUNCTORS ARE LINEARLY ORDERED

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Valuation domains have been extended in the non-commutative case by several authors, giving rise to the so called generalized valuation rings, that is, rings whose lattice of right ideals is linearly ordered by inclusion. We propose here the study of rings whose lattice of kernel functors is linearly ordered and we indicate throughout this article similarities between them and valuation rings. In addition, the rings presented here include generalized valuation rings and coincide with them when commutativity is assumed. They therefore provide a new non-commutative analogue of valuation rings.

Unlike the generalized valuation rings, the rings we study enjoy properties that transfer nicely to matrix rings thus enabling us to treat questions in a broader context.

Finally, a semigroup structure imposed on the lattice of kernel functors is analyzed and the article concludes by examining when that semigroup can be thought of as the semigroup of a valuation ring.

Preliminaries. All rings occurring are associative and possess unity, which is preserved under subrings and ring homomorphisms. Unless otherwise stated all modules are unitary right modules. We let $\mathscr{M}_R$ denote the category of right R-modules.

For any module M we let $E(M)$ stand for an injective hull of M. Hence M is large in $E(M)$ and $E(M)$ is injective. If M is a module, N a submodule of M and S a nonempty subset of M we let $(N \underset{R}{:} S)$ denote the right ideal $\{r \in R; Sr \subset N\}$. When no danger of confusion arises we will simply write $(N : S)$.

The term ideal is reserved to be used for two-sided ideals only. Consequently, a ring is simple if it has exactly two ideals.

Notation and terminology concerning kernel functors, (topologizing) filters of right ideals, etc., will follow Goldman [**4**] and Stenstrom [**9**] with which familiarity is assumed.

The class of all objects of $\mathscr{M}_R$ which are torsion with respect to a given kernel functor is closed under taking submodules, homomorphic images and arbitrary direct sums, conditions that characterize what we will call a torsion class throughout this paper. There is a one to one correspondence between kernel functors, filters of right ideals and torsion classes.