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AN INVARIANCE PRINCIPLE FOR ASSOCIATED RANDOM FIELDS

ROBERT M. BURTON, JR. AND TAE-SUNG KIM

Applying known tightness criteria to Poisson cluster random measures, it is shown that if the total member size has a finite $2+\delta$ moment, then the random measure satisfies an invariance principle.

I. Introduction. Let $\{X_{\underline{k}} \mid \underline{k} \in \mathbf{Z}^d\}$ be a random field that is centered, stationary, associated and has a summable covariance function. C. Newman **[10]** showed that, when viewed as an element in d-dimensional Skorohod space, the renormalizations of $\{X_{\underline{k}} \mid \underline{k} \in \mathbf{Z}^d\}$ converge to a Wiener measure in the sense of finite dimensional distributions. Newman and Wright **[11]** showed that this may be improved to an invariance principle if $d = 1$ or 2. Analogous results hold in the case of random measures. A tightness criterion of Bickel and Wichera **[1]** is applicable in the case of general d. This criterion is applied to Poisson center cluster random measures. It is shown that if the total member size has a finite $2+\delta$ moment then the random measure satisfies an invariance principle.

II. Random fields and random measures. A *random field* is a collection of nondegenerate random variables indexed by $\mathbf{Z}^d$ and is denoted $\{X_{\underline{k}} \mid \underline{k} \in \mathbf{Z}^d\}$. All random fields in this section are assumed centered and stationary, i.e. $E[X_{\underline{k}}] = 0$ and the distribution is invariant with respect to translations of the indices by the group $\mathbf{Z}^d$. A random field is *associated* if whenever $A \subseteq \mathbf{Z}^d$ is a finite subset and $f, g\colon \mathbf{R}^A \to \mathbf{R}$ are coordinatewise increasing then $\operatorname{Cov}[f(X_{\underline{k}}\colon \underline{k} \in A), g(X_{\underline{k}}\colon \underline{k} \in A)]$ is nonnegative whenever the covariance is defined. Association is a strong positive dependence property implying, in particular, nonnegative correlations of the random variables $X_{\underline{k}}$ (if they exist). For details concerning association see Esary, Proschan and Walkup **[4]**.

A random field may be interpolated and rescaled to form a random element of d-dimensional Skorohod space $\mathfrak{D}([0,1]^d)$ by setting

$$W_n(\underline{t}) = n^{-d/2} \sum_{j_1=1}^{[nt_1]} \cdots \sum_{j_d=1}^{[nt_d]} X_{\underline{j}}$$