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FULL ANALYTIC SUBSPACES FOR CONTRACTIONS WITH RICH SPECTRUM

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It is shown that contractions with sufficiently rich spectrum have special kinds of invariant subspaces. These subspaces are analogous to the Hardy space H^2 when viewed as an invariant subspace for the contraction of multiplication by z on L^2 (of the circle).

1. Introduction. This paper deals with the invariant subspace structure of certain contractions on a separable Hilbert space $\mathscr{H}$. If S is an operator on $\mathscr{H}$ of norm one, then the left essential spectrum of S, $\sigma_{\mathrm{le}}(S)$, is called dominating in the unit disc if almost every point of the unit circle (with resepct to arc length measure) is a nontangential limit of a sequence from $\sigma_{\mathrm{le}}(S)$. Throughout this paper, it will be assumed that S is a completely nonunitary contraction (i.e., $\|S\| \leq 1$) with $\sigma_{\mathrm{le}}(S)$ dominating in the unit disc. A closed invariant subspace, $\mathscr{S}$, for S will be called full analytic, if S and $\mathscr{S}$ together satisfy two properties. The first is that $\mathscr{S}$ is vector space isomorphic to a subspace of the vector space, $H(U)$, of all functions analytic on the open unit disc, U. The second is that if $y \in \mathscr{S}$ is associated with $f_y(z) \in H(U)$ under this isomorphism, then Sy is associated with $zf_y(z) \in H(U)$, so $f_{Sy}(z) = zf_y(z)$. The main result of this paper, Theorem 2, gives that S has a closed invariant subspace that is full analytic.

The phenomenon of full analytic invariant subspaces occurs frequently in analysis. The classical example is displayed by the bilateral shift and the Hardy space H^2. Even more generally, every subnormal operator has a full analytic subspace as was shown recently by Olin and Thomson in [7] (the domain of analyticity has to be appropriately altered). One can find a simpler proof of this result for just normal operators in Chapter IX of [3]. The result of this present paper, although it does not subsume the subnormal or measure theoretic results concerning analyticity and vice versa, was clearly suggested by the work of Olin and Thomson.

The main tool to be used is that of matricial factorization as developed by Bercovici, Foias, and Pearcy in [1]. They used these results together with J. Langsam [2] to prove (among other things) that the