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SHIFTS OF INTEGER INDEX ON THE HYPERFINITE II_1 FACTOR

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In this paper we consider shifts on the hyperfinite II_1 factor arising as a generalization of a construction of Powers. We determine the conjugacy classes of certain of these shifts.

1. Introduction. Let R be the hyperfinite II_1 factor with normalized trace tr. A shift α on R is an identity-preserving $*$-endomorphism which satisfies $\bigcap_{m\geq 1} \alpha^m(R) = \mathbf{C}1$. We say that α has shift index n if the subfactor $\alpha(R)$ has the same index $n = [R: \alpha(R)]$ in R as defined by Jones, in **[2]**.

In **[3]** Powers considered shifts of index 2 on R. These were constructed using functions $\sigma: \mathbf{N} \cup \{0\} \to \{-1, 1\}$ and sequences $\{u_j: j \in \mathbf{N}\}$ of self-adjoint unitaries satisfying $u_i u_j = \sigma(|i-j|) u_j u_i$. If $A(\sigma)$ is the $*$-algebra generated by the $\{u_j\}$ and tr is the normalized trace on $A(\sigma)$ defined by $\mathrm{tr}(w) = 0$ for any non-trivial word in the u_i, the GNS construction $(\pi_{\mathrm{tr}}, H_{\mathrm{tr}}, \Omega_{\mathrm{tr}})$ gives rise to the von Neumann algebra $M = \pi_{\mathrm{tr}}(A(\sigma))''$. Different characterizations were given in **[3]** and **[4]** for M to be the hyperfinite factor R. In **[4]** it was shown this is the case if and only if the sequence $\{\dots, \sigma(2), \sigma(1), \sigma(0), \sigma(1), \sigma(2), \dots\}$ is aperiodic. For this case, the shift α on $M = R$ defined by the relations $\alpha(\pi_{\mathrm{tr}}(u_i)) = \pi_{\mathrm{tr}}(u_{i+1})$ has index 2. In **[3]** it was shown that the σ-sequence above is a complete conjugacy invariant for α. (We say shifts α, β are conjugate if there exists an automorphism γ of R such that $\alpha = \gamma \cdot \beta \cdot \gamma^{-1}$.)

Motivated by **[3]**, Choda in **[1]** considered shifts of index n, defined on R by $\alpha(u_j) = u_{j+1}$, for a sequence of unitaries $\{u_j\}$ generating R, and satisfying $(u_j)^n = 1, u_1 u_{j+1} = \sigma(j) u_{j+1} u_1$, where $\sigma: \mathbf{N} \cup \{0\} \to \{1, \exp(2\pi i/n)\}$. In this setting and under the assumption $\alpha(R)' \cap R = \mathbf{C}1$ she characterizes the normalizer $N(\alpha)$ of α (see Definition 3.4) and the unitary α-generators of R.

In this paper we generalize some of the results of **[1,3,4]**. In §2 we consider, for a fixed n, algebras generated by sequences $\{u_j\}$ of unitaries, of order n, and satisfying $u_1 u_{j+1} = \sigma(j) u_{j+1} u_1$ for functions $\sigma: \mathbf{N} \cup \{0\} \to \Omega_n$, the set of nth roots of unity. We determine