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MIXED CUSP FORMS AND HOLOMORPHIC FORMS ON ELLIPTIC VARIETIES

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Let E^m be an elliptic variety. We define mixed cusp forms associated to E^m and prove that the space of mixed cusp forms coincides with the space of holomorphic $(m+1)$-forms on E^m. We also determine the geometric genus of E^m under certain conditions.

0. Introduction. Let $\pi\colon E \to X$ be an elliptic fibration and E_0 the union of its regular fibers. The nonsingular variety E^m obtained by resolving the singularities of the compactification of the fiber product

$$\overbrace{E_0 \times_\pi E_0 \times_\pi \cdots \times_\pi E_0}^{m}$$

is called an elliptic variety.

In a series of papers ([7], [8], [9]) Shokurov has constructed elliptic varieties and proved several properties of Kuga's modular varieties which are elliptic varieties of a special kind. If $m = 1$, E^m is simply an elliptic surface and a Kuga's modular variety is an elliptic modular surface of Shioda ([6]).

Hunt and Meyer ([3]) have introduced mixed cusp forms associated to an elliptic surface. They have shown that the space of mixed cusp forms coincides with the space of holomorphic 2-forms and used this fact to determine the geometric genus of the elliptic surface.

The main purpose of this paper is to extend the definition of mixed cusp forms of Hunt and Meyer to the case of elliptic varieties and determine the geometric genus of E^m.

1. Elliptic surfaces. Let E be an elliptic surface with a global section over its base curve in the sense of Kodaira ([4]). Thus E is the total space of an elliptic fibration $\pi\colon E \to X$ over a Riemann surface X with a section $s\colon X \to E$ such that the generic fiber of π is an elliptic curve.

Let E_0 be the union of the regular fibers of π and let $X_0 = \pi(E_0)$. The universal cover of X_0 is the Poincaré upper half plane $\mathfrak{h}$. Let $G \subset \mathrm{PSL}(2, \mathbf{R})$ be a fuchsian group acting on $\mathfrak{h}$ by linear transformations such that $X = G\backslash\mathfrak{h}^*$ where $\mathfrak{h}^* = \mathfrak{h} \cup \{G\text{-cusps}\}$. Since $G = \pi_1(X_0)$,