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ACCELERATION BY SUBSEQUENCE TRANSFORMATIONS

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The acceleration field of subsequence matrix transformations are studied with respect to the convergence rate of the sequence being accelerated. Included is a proof that no subsequencing algorithm exists which will determine a set of subscripts $(n(i))$ for which $(y_{n(i)})$ will be linear for every y which converges at the same rate as or faster (slower) than a fixed sequence x.

1. Introduction. D. F. Dawson [2] has characterized the summability field of a matrix A by showing A is convergence preserving over the set of all sequences which converge faster than some fixed sequence x, A is convergence preserving over the set of all sequences, or A only preserves the limit of a set of constant sequences. We seek an analog to this result dealing with the acceleration field of a subsequence transformation.

The sequence x converges to σ faster than the sequence y converges to λ $(x < y)$ if

$$\lim_n (x_n - \sigma)/(y_n - \lambda) = 0.$$

(In this case we also say that y converges to λ slower than x converges to σ.) The matrix $A = (a_{pq})$ accelerates the convergence of x if $Ax < x$. The acceleration field of A is $\{x\colon Ax < x\}$. The sequence x converges to σ at the same rate as the sequence y converges to λ $(x \approx y)$ if

$$0 < \varliminf_n |(x_n - \sigma)/(y_n - \lambda)| \le \varlimsup_n |(x_n - \sigma)/(y_n - \lambda)| < +\infty.$$

In §2 below, the basic background for investigating the acceleration field of a subsequence transformation in terms of rate of convergence is presented. The possibility of obtaining an analog to Dawson's result for the acceleration field of a subsequence transformation is considered in §3. Subsequences have been used by C. Brezinski, J. P. Delahaye, and B. Germain-Bonne [1] to generate an acceleration algorithm for a restricted class of sequences. In §4 it is shown that this algorithm cannot be extended to a larger class of sequences defined in terms of rate of convergence.