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ON THE SATO-SEGAL-WILSON SOLUTIONS OF THE K-dV EQUATION

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We discuss the class of solutions of the K-dV equation found by Sato, Segal, and Wilson. We relate this class of solutions to properties of the Weyl *m*-functions, and of the Floquet exponent for the random Schrödinger equation.

1. Introduction. In a series of recent papers, Date, Jimbo, Kashiwara, and Miwa [**5, 6, 7, 8, 9**] have developed ideas of M. and Y. Sato [**23, 24**] for finding solutions of the Kadomtsev-Petviashvili (K-P) hierarchy. The solutions of the K-P hierarchy discussed in these papers are expressed in terms of the so-called τ-function, which can be viewed as a generalization of the Riemann Θ-function.

Even more recently, Segal and Wilson [**25**] have given a careful formulation of the work of the Kyoto group. A consequence of their analysis is the following. Recall that one equation of the K-P hierarchy is the Korteweg-de Vries (K-dV) equation:

$$\frac{\partial u}{\partial t} = 6u\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial x^3}, \qquad u(0, x) = u_0(x), \tag{1}$$

viewed as an evolution equation with initial data $u_0(x)$. Segal and Wilson produce a class $\mathscr{C}^{(2)}$ of initial conditions (or "potentials") $u_0(x)$ for which (1) admits a solution $u(t, x)$ which is meromorphic in t and x. The class $\mathscr{C}^{(2)}$ contains the solitons (see, e.g., [**1**]) and the algebro-geometric potentials [**11, 18, 21**]. We will call the elements of $\mathscr{C}^{(2)}$ Sato-Segal-Wilson potentials.

The purpose of the present note is to describe in some detail a subclass LP (for "limit-point"; see below) of the class $\mathscr{C}^{(2)}$. Namely, consider the Schrödinger equation

$$L\phi = \left(\frac{-d^2}{dx^2} + u_0(x)\right)\phi = \lambda\phi \tag{2}$$

with potential $u_0(x)$. Define LP $\subset \mathscr{C}^{(2)}$ to be the set of Sato-Segal-Wilson potentials which are real and finite for all real x, and for which L is in the limit-point case $x = \pm\infty$ ([**26**]; [**3**, Ch. 9]). Let $m_+(\lambda)$ be the