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STABILITY OF UNFOLDINGS IN THE CONTEXT OF EQUIVARIANT CONTACT-EQUIVALENCE

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M. Golubitsky and D. Schaeffer introduced the notion of equivariant contact-equivalence between germs of C^∞ equivariant mappings, in order to study perturbed bifurcation problems having a certain symmetry property. The main tool used is the so-called "Unfolding Theorem" for the qualitative description of the symmetry-preserving perturbations of these problems. From the point of view of applications, a relevant notion is that of stability of unfoldings. In this paper we prove the equivalence of the universality and the stability of unfoldings in the context of equivariant contact-equivalence.

1. Universal Γ-unfolding. Let Γ be a compact Lie group acting orthogonally on $\mathbf{R}^n$ and $\mathbf{R}^p$. We write $\mathscr{E}^{\Gamma}_{n,p}$ for the space of C^∞ germs $f\colon (\mathbf{R}^n, 0) \to \mathbf{R}^p$ of Γ-equivariant mappings (i.e. $f(\gamma x) = \gamma f(x)$ for all $\gamma \in \Gamma$). The space of Γ-invariant C^∞-germs $h\colon (\mathbf{R}^n, 0) \to \mathbf{R}$ (i.e. $h(\gamma x) = h(x)$ for all $\gamma \in \Gamma$) is denoted by $\mathscr{E}^{\Gamma}_n$. In what follows we shall consider germs $G\colon (\mathbf{R}^n \times \mathbf{R}, 0) \to \mathbf{R}^p$ and $F\colon (\mathbf{R}^n \times \mathbf{R} \times \mathbf{R}^q, 0) \to \mathbf{R}^p$ and we shall assume that Γ acts trivially on $\mathbf{R}$ and $\mathbf{R}^q$.

The notion of equivariant contact-equivalence introduced by Golubitsky and Schaeffer [3] is the following:

DEFINITION 1.1. We say that G_1 and $G_2 \in \mathscr{E}^{\Gamma}_{n+1,p}$ are Γ-equivalent if

$$G_1(x, \lambda) = T(x, \lambda) G_2(X(x, \lambda), \Lambda(\lambda))$$

where

$$(1.1.1) \qquad T\colon (\mathbf{R}^n \times \mathbf{R}, 0) \to \mathrm{Gl}_p(\mathbf{R}) \quad \text{is } C^\infty.$$

$$(1.1.2) \qquad (X, \Lambda)\colon\ (\mathbf{R}^n \times \mathbf{R}, 0) \to (\mathbf{R}^n \times \mathbf{R}, 0) \quad \text{is } C^\infty,$$

$$\det(d_x X(0)) > 0 \quad \text{and} \quad \Lambda'(0) > 0.$$

$$(1.1.3) \qquad X(\gamma x, \lambda) = \gamma X(x, \lambda) \quad \text{for all } \gamma \in \Gamma.$$