PACIFIC JOURNAL OF MATHEMATICS
Vol. 132, No. 2, 1988

ON DEFORMING G-MAPS TO BE FIXED POINT FREE

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When $f: M \to M$ is a self-map of a compact manifold and $\dim M \geq 3$, a classical theorem of Wecken states that f is homotopic to a fixed point free map if, and only if, the Nielsen number $n(f)$ of f is zero. When M is simply connected, and $\dim M \geq 3$ the NASC becomes $L(f) = 0$, where $L(f)$ is the Lefschetz number of f. An equivariant version of the latter result for G-maps $f: M \to M$, where M is a compact G-manifold, is due to D. Wilczyński, under the assumption that M^H is simply connected of dimension ≥ 3 for any isotropy subgroup H with finite Weyl group WH. Under these assumptions, f is G-homotopic to a fixed point free map if, and only if, $L(f^H) = 0$ for any isotropy subgroup H (WH finite), where $f^H = f|M^H$ and M^H represents those elements of M fixed by H. A special case of this result was also obtained independently by A. Vidal via equivariant obstruction theory. In this note we prove the analogous equivariant result without assuming that the M^H are simply connected, assuming that $n(f^H) = 0$, for all H with WH finite. There is also a codimension condition. Here is the main result.

THEOREM. *Let G denote a compact Lie group and M a compact, smooth G-manifold. Let $(H_1), \dots, (H_k)$ denote an admissible ordering of the isotropy types of M, $M_i = \{x \in M: (G_x) = (H_j),\ j \leq i\}$ the associated filtration. Also, let $\mathcal{F}$ denote the set of integers i, $1 \leq i \leq k$, such that the Weyl group $WH_i = NH_i/H_i$ is finite. Suppose that for each $i \in \mathcal{F}$, $\dim M^{H_i} \geq 3$ and the codimension of $M_{i-1} \cap M^{H_i}$ in M^{H_i} is at least 2. Then, a G-map $f: M \to M$ is G-homotopic to a fixed point free G-map $f': M \to M$ if, and only if, the Nielsen number $n(f^{H_i}) = 0$ for each $i \in \mathcal{F}$.*

1. Preliminaries. Throughout this note G will denote a compact Lie group and M will denote a compact, smooth G-manifold. For any closed subgroup H in G, we denote by NH the normalizer of H in G and by $WH = NH/H$, the Weyl group of H in G. The conjugacy class of H, denoted by (H), is called the orbit type of H. If $x \in M$ then G_x denotes the isotropy subgroup of x, i.e. $G_x = \{g \in G | gx = x\}$. For each subgroup H of G, $M^H = \{x \in M | hx = x \text{ for all } h \in H\}$ and $M_H = \{x \in M | G_x = H\}$. Let $\{(H_j)\}$ denote the (finite) set of isotropy