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ORTHOGONAL BASES ARE SCHAUDER BASES AND A CHARACTERIZATION OF Φ-ALGEBRAS

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It is shown that every orthogonal basis in a topological algebra is a Schauder basis. A new class of topological algebras called Φ-algebras is introduced. They are characterized in terms of seminorms. Among other results, a necessary and sufficient condition for a locally convex *s*-algebra with an unconditional orthogonal basis to be an *A*-convex algebra is given.

0. Introduction. One of the interesting questions in the classical basis theory is whether every basis in a topological vector space of a certain type is a Schauder basis. Positive answers to this question have been established in the situations where the open mapping theorem holds, for instance, when the topological vector space is an F-space (Arsove [**1**]), in particular, a Banach space.

In the context of topological algebras, the second author and Watson showed that an orthogonal basis in a locally m-convex algebra (a setting where the open mapping theorem does not necessarily hold) is always a Schauder basis ([7], Proposition 3.1). Here we establish this result in its ultimate general setting; indeed, we show that every orthogonal basis in a topological algebra is a Schauder basis (Theorem 1.1).

Improving upon a result of Husain and Watson [7], we show that every locally m-convex algebra with an identity and an orthogonal basis is topologically isomorphic with a dense subalgebra of the algebra s of all complex sequences with the pointwise operations and the topology of pointwise convergence (Theorem 2.1).

In §3, we introduce a class of topological algebras with orthogonal basis which we call Φ-algebras. Examples of such algebras are given and a characterization of Φ-algebras in terms of seminorms is proved (Theorem 3.3).

In §4, we study different types of topological algebras with orthogonal bases. Among other results, we give a necessary and sufficient