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ON MATRICIALLY NORMED SPACES

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Arveson and Wittstock have proved a "non-commutative Hahn-Banach Theorem" for completely bounded operator-valued maps on spaces of operators. In this paper it is shown that if T is a linear map from the dual of an operator space into a C^*-algebra, then the usual operator norm of T coincides with the completely bounded norm. This is used to prove that the Arveson-Wittstock theorem does not generalize to "matricially normed spaces". An elementary proof of the Arveson-Wittstock result is presented. Finally a simple bimodule interpretation is given for the "Haagerup" and "matricial" tensor products of matricially normed spaces.

1. Introduction. A *function space* V on a set X is a linear subspace of the bounded complex functions on X. With the uniform norm, this is a normed vector space. Conversely, any (complex) normed vector space V may be realized as a function space on the closed unit ball X of the dual space V^*. Thus one may regard a normed vector space as simply an abstract function space.

An *operator space* V on a Hilbert space H is a linear subspace of the bounded operators on H. For each $n \in \mathbf{N}$, the operator norm associated with H^n determines a distinguished norm on the $n \times n$ matrices over V. The second author recently gave an abstract characterization for the operator spaces by taking into consideration these systems of matrix norms. The operator spaces V are characterized among the "matricially normed spaces" (see §2), by the "L^∞-property": given matrices $v = [v_{ij}]$, $w = [w_{kl}]$ with $v_{ij}, w_{kl} \in V$,

$$\|v \oplus w\| = \max\{\|v\|, \|w\|\}.$$

On the other hand, the dual of an operator space is canonically an "L^1-matricially normed space", in the sense that its matrix norms satisfy

$$\|v \oplus w\| = \|v\| + \|w\|.$$

In this paper we shall begin a systematic study of the matricially normed spaces. Our main results are:

(a) We show in §2 that if $\varphi: V \to W$ is a linear map from an L^1-matricially normed space to an operator space, then the completely