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A NOTE ON DERIVATIONS WITH POWER CENTRAL VALUES ON A LIE IDEAL

JEFFREY BERGEN AND LUISA CARINI

Let R be a prime ring of characteristic $\neq 2$ with a derivation $d \neq 0$ and a non-central Lie ideal U such that $d(u)^n$ is central, for all $u \in U$. We prove that R must satisfy s_4, the standard identity in 4 variables; hence R is either commutative or an order in a 4-dimensional simple algebra. This result extends a theorem of Herstein to Lie ideals.

In [**2**] Herstein shows that if R is a prime ring with center Z and if $d \neq 0$ is a derivation of R such that $d(x)^n \in Z$, for all $x \in R$, then R satisfies s_4, the standard identity in 4 variables. This theorem indicates that the global structure of a ring is often tightly connected to the behaviour of one of its derivations. The purpose of this note is to show that the same conclusion holds for prime rings of characteristic $\neq 2$ even if we assume only that $d(u)^n$ is central for those u in some non-central Lie ideal.

We will proceed by first proving the result when d is inner. We will then use Kharchenko's theorem on differential identities [**5**] to reduce to the case where d is inner on the Martindale quotient ring of R. By using Kharchenko's theorem, our proof will actually be somewhat simpler than the proof in [**2**].

In all that follows, unless stated otherwise, R will be a prime ring of characteristic $\neq 2$, U a non-central Lie ideal of R, $d \neq 0$ a derivation of R, and $n \geq 1$ a fixed integer such that $d(u)^n$ is central, for all $u \in U$. For any ring S, $Z = Z(S)$ will denote its center. For subsets $A, B \subset R$, $[A, B]$ will be the additive subgroup generated by all $[a, b] = ab - ba$; $a \in A$, $b \in B$. In addition, s_4 will denote the standard identity in 4 variables.

By a result of Herstein [**3**], $U \supset [I, R]$ for some $I \neq 0$, an ideal of R. Therefore, we will assume throughout that $U \supset [I, R]$.

We will also make frequent and important use of the following three results. We do not necessarily state them in their fullest generality.

1. (Carini-Giambruno [**1**].) If $U \not\subset Z(R)$ is a Lie ideal of a prime ring R of characteristic $\neq 2$, and if d is a derivation of R such that $d(u)^n = 0$, for all $u \in U$, then $d = 0$.